University of Illinois **Spring 2025**

ECE 313: Problem Set 9

Due: Sunday, April 13 at 11:59 p.m.
Reading: *ECE 313 Course Notes,* Sections 3.8, 3.10, 4.1

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. **This homework is instead due by 11:59 p.m. on the following Sunday. Please write down your work and derivations. An answer without justification as of how it is found will not be accepted.** You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.
Please write on the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned page numbers.**

1. **[Function of a RV 1]**
   Let X be a uniform random variable with support $[-1, 2]$ and let $Y = X + |X|$. Find the CDF of Y.

2. **[Function of a RV 2]**
   Let X be a exponential random variable with parameter $\lambda = 1$. Let $Y = 1/X$. Find the pdf of Y.

3. **[Generating a Weibull Distribution]**
   Let X be uniformly distributed on $(0, 1)$, and let $Y = g(X)$, where $g(\cdot)$ is a function of X. We want Y to have the CDF of a Weibull distribution with shape parameter $\beta > 0$ and scale parameter $\alpha > 0$; that is, $F_Y(v) = 1 - e^{-\left(\frac{v}{\alpha}\right)^{\beta}}$ for $v \geq 0$, and zero otherwise. Find a function $g(\cdot)$ to accomplish this, and check that this indeed gives the desired distribution.

4. **[Binary Hypothesis Testing 1]**
   Consider the following binary hypothesis testing problem. Under H_0, the random variable X has the pdf f_0, while under H_1, the random variable X has the pdf f_1, where

$$f_0(u) = \begin{cases} \frac{1}{4} & u \in \left[\frac{-1}{2}, \frac{3}{2}\right] \cup \left[\frac{5}{2}, \frac{9}{2}\right], \\ 0 & \text{else} \end{cases}$$

and

$$f_1(u) = \begin{cases} \frac{1}{4}u & u \in [0,2], \\ \frac{-1}{4}u + 1 & u \in (2,4], \\ 0 & \text{else} \end{cases}$$

Assume that $4\pi_0 = \pi_1$.

(a) Find the ML rule.

(b) Find $p_{false\,alarm}$, p_{miss}, and p_e for the ML rule.

(c) Find the MAP rule.

(d) Find $p_{false\,alarm}$, p_{miss}, and p_e for the MAP rule.

5. **[Binary Hypothesis Testing 2]**
Consider a binary hypothesis testing problem where the observation X is exponentially distributed with parameter λ under H_0 and uniformly distributed in $[0, b]$ under H_1.

   (a) Find the value(s) of π_0 (the prior probability H_0 is true) such that the MAP decision rule would always select H_0.

   (b) Find the probability of error p_e for the MAP decision rule under the above condition.

6. **[Joint CDFs]**
Consider the following function:

$$F(u, v) = \begin{cases} 0, & u + v \leq 1, \\ 1, & u + v > 1. \end{cases}$$

Is this a valid joint CDF? Why or why not? Prove your answer and show your work.