

ECE 313: Problem Set 5

Due: Friday, Oct. 9 at 06:59:00 p.m.

Reading: *ECE 313 Course Notes, Section 2.11, 3.1.*

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.

Please write on the top right corner of the first page:

NAME

NETID

SECTION

PROBLEM SET #

Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned page numbers.**

1. **[Detection Problem with the geometric distribution]**

The number of attempts, Y , required for a certain basketball player to make a 25 foot shot, is observed, in order to choose one of the following two hypotheses:

H_1 (outstanding player) : Y has the geometric distribution with parameter $p = 0.6$,

H_0 (average player) : Y has the geometric distribution with parameter $p = 0.25$.

- (a) Describe the ML decision rule. Express it as directly in terms of Y as possible.
- (b) Find $p_{\text{false alarm}}$ and p_{miss} for the ML rule.
- (c) Describe the MAP decision rule under the assumption that H_0 is a priori twice as likely as H_1 . Express the rule as directly in terms of Y as possible.
- (d) Find the average error probability, p_e , for both the ML rule and the MAP rule, using the same prior distribution given in part (c). For which rule is the average error probability smaller?

2. **[Testing hypotheses about a die]**

Consider a binary hypothesis testing problem based on observation of n independent rolls of a die. Let $X_1, \dots, X_n$ denote the numbers rolled on the die. Let a denote a known constant that is slightly greater than one. The two hypotheses are:

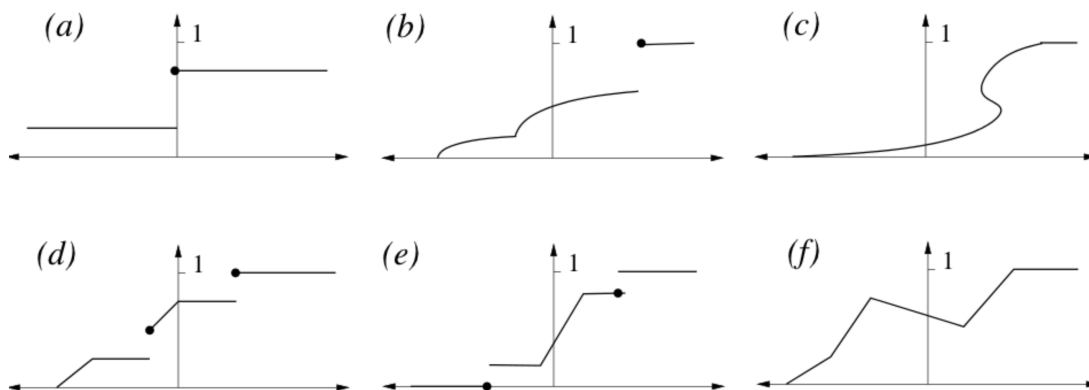
H_0 : the die is fair

H_1 : for each roll of the die, i shows with probability $p_i = Ca^i$, where $C = \frac{1-a}{a-a^n}$, so that the probabilities sum to one.

- (a) Find simple expressions for $p_i(u_1, \dots, u_n) = P(X_1 = u_1, \dots, X_n = u_n \mid H_i)$ for $i = 0$ and $i = 1$, where $u_i \in \{1, 2, 3, 4, 5, 6\}$ for each i . Express your answers using the variables t_k , for $1 \leq k \leq 6$, where t_k is the number of the n rolls that show k . The vector $(t_1, \dots, t_6)$ is called the type vector of $(u_1, \dots, u_n)$. Intuitively, the order of the observations shouldn't matter, so decision rules will naturally only depend on the type vector of the observation sequence.
- (b) Find a simple expression for the likelihood ratio, $\Lambda(u_1, \dots, u_n) = \frac{p_1(u_1, \dots, u_n)}{p_0(u_1, \dots, u_n)}$, and describe, as simply as possible, the likelihood ratio test for H_1 vs. H_0 given the observations $(u_1, \dots, u_n)$.
- (c) In particular, suppose that $n = 100$, $(t_1, \dots, t_6) = (18, 12, 13, 19, 18, 20)$, and $a = 1.1$. Which hypothesis does the maximum likelihood decision rule select?

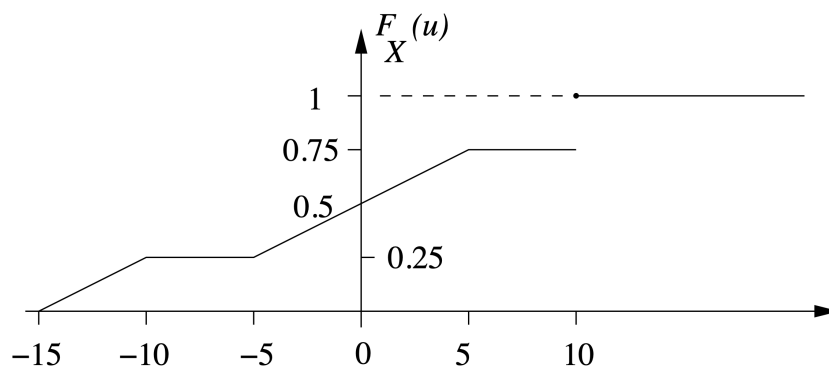
3. [CDFs]

Which of the six plots below show valid CDFs? For each one that is not valid, state a property of CDFs that is violated.



4. [CDFs]

Let X be a random variable with the CDF shown.



Compute the following probabilities:

- (a) $P\{X \leq -12\}$
 (b) $P\{-12 < X < 4\}$
 (c) $P\{X > 5\}$

- (d) $P\{X \geq 10\}$
- (e) $P\{|X| \leq 5\}$
- (f) $P\{X^2 \geq 100\}$