

ECE 313: Problem Set 3

Due: Friday, September 25 at 06:59:00 p.m.

Reading: *ECE 313 Course Notes, Section 2.3, 2.10, 2.4.1-2.4.2*

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.

Please write on the top right corner of the first page:

NAME

NETID

SECTION

PROBLEM SET #

Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned page numbers.**

1. **[Conditional probability and the binomial distribution]**

Let X denote the number of heads observed in 12 tosses of a fair coin.

- (a) Find $P\{X \leq 6 \mid X \geq 5\}$.
- (b) Given that $X = 5$, what is the (conditional) probability that the 5th toss was a head?

2. **[Independence]**

Suppose two dice, one orange and one blue, are rolled. Define the following events:

A : The product of the two numbers that show is 8.

B : The number on the orange die is strictly larger than the number on the blue die.

C : The sum of the numbers is divisible by four.

D : The number on the orange die is either 2 or 5.

- (a) List all pairs of events from the set A, B, C, D that are independent.
- (b) List all triplets of events, if any, from the set of A, B, C, D , that are mutually independent.

3. **[Binomial distribution]**

Six cars start out on a cross-country race. The probability that a car breaks down and drops out of the race is 0.1. Cars break down independently of each other.

- (a) What is the probability that exactly three cars finish the race?
- (b) What is the probability that at most three cars finish the race?
- (c) What is the probability that at least four cars finish the race?

4. **[Geometric Distribution]**

Let X be a geometric random variable with parameter $p = 1/3$, and let $Y = \cos\left(\frac{\pi X}{2}\right)$. Find the pmf of Y in closed-form.

5. **[Negative Binomial Distribution]**

Let X be a random variable representing the number of times one must throw a red die until the outcome 2 has occurred five times. Let Y be a random variable representing the number of times one must throw a blue die until the outcomes 1 or 2 have occurred five times.

(a) Find the pmf, the expected value and the standard deviation of X .

(b) Find the pmf, the expected value, and the standard deviation of Y .

6. **[Flipping a coin]**

Suppose that a fair coin is independently tossed twice. Define the events:

$$A = \{\text{"The first toss is a head"}\}$$

$$B = \{\text{"The second toss is a head"}\}$$

$$C = \{\text{"Both tosses yield the same outcome"}\}$$

Are A, B, C independent? Please show your work.