Caveats

- Problems in the slide will be independent from midterm problems
  - $P(p_1 | p_1' \text{ in slide}) = P(p_1)$
- All numbers will be replaced by symbols in the slide
  - In midterm, you may need to compute
- We will cover top-K options from the Slido survey
  - Survey does not cover all topics
  - You still need to review all topics by yourself

Agenda & Survey result

- Poisson Process
  - Exponential Distribution
- Scaling of PDF
- Markov and Chebyshev
- PDF/CDF

Bernoulli Process $\xrightarrow{h \to 0,\ \lambda = \frac{p}{h}}$ Poisson Process

- Assume each trial takes h duration to complete

$Bern(p)$

$Bin\left(n = \left\lfloor \frac{t}{h} \right\rfloor, p = \lambda h\right)$

$Geo(p = \lambda h)$

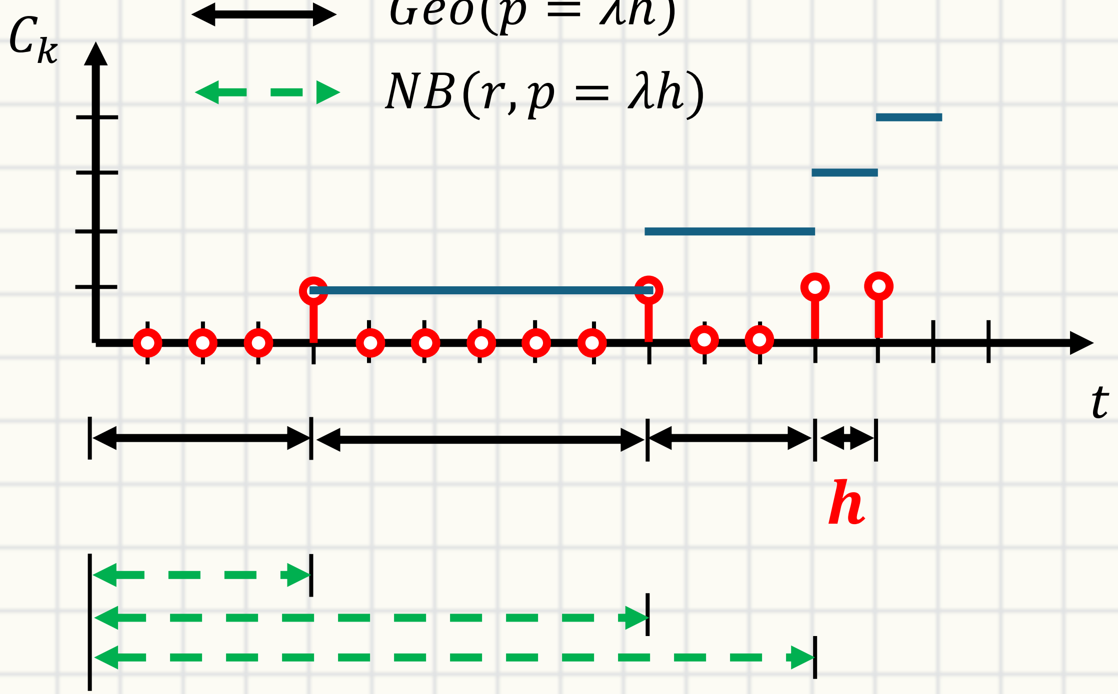
$NB(r, p = \lambda h)$

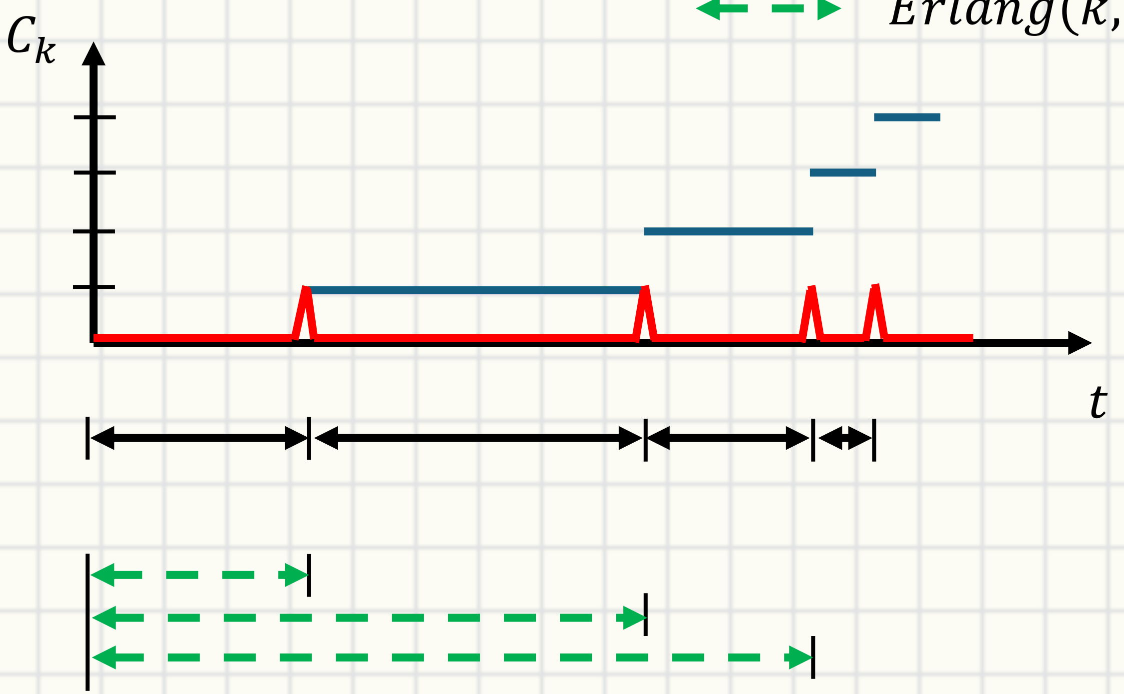
$$\xrightarrow{h \to 0,\ \boxed{\lambda} = \frac{p}{h} = \left(\frac{P(Head)}{Time}\right)}$$

$Poi(\lambda = \lambda t)$

$Exp(\lambda)$

$Erlang(k, \lambda)$





Properties

	$Exp(\lambda)$	$Poi(\lambda = \lambda_{ref} t)$
Mean		
Variance		
PDF/ PMF		
CDF		
Example	System lifetime	Event occurrence within t
Special		

Poisson Process

Poison RV

- $\lambda = \lambda_{ref} t$
- None-overlapped process are independent.

(RV)

A support center is receiving λ call/ mins. Probability of

- Exactly 4 calls in 2 mins
- At least 3 calls in 1 min
- 5 calls in 5 mins, given 2 calls in the third mins
- 5 calls in 5 mins, among which 2 calls in the third min

Scaling of PDF

$X = \frac{Y-b}{a}$

expand X support by a, height (pdf) ↓ a.

Let $Y = aX + b$, where X, Y are RV and a, b are constants

- $f_Y(u) = f_X\left(\frac{u-b}{a}\right) \times \frac{1}{|a|}$

$$f_X(u) = \begin{cases} 1 & -0.5 \le u \le 0 \\ 1-u & 0 \le u \le 1 \end{cases}$$, let $Y = 2X + 1$

- Solve $E[Y]$ and σ_Y^2
- $P\{Y \ge 2\}$

① Back to X

$E[Y] = 2E[X] + 1$

$E[X] \to E[Y]$

$P\{2X+1 \ge 2\} = P\{X \ge 0.5\}$

② $f_Y(c) = f_X\left(\frac{c-\cancel{b}^{\,1}}{\cancel{a}_{\,2}}\right)\frac{1}{|a|} = f_X\left(\frac{c-1}{2}\right)\times\frac{1}{|2|}$

$$f_Y(c) = \begin{cases} \frac{1}{2} & 0 \le c \le 1 \\ & 1 \le c \le 3. \end{cases}$$

$$\frac{1-\left(\frac{c-1}{2}\right)}{|2|} = \boxed{\frac{3-c}{4}}$$

Markov and Chebyshev

$= \int_{0.5}^{1} f_X(u)\,du.$

Markov

- Check whether Y is none-negative
- Compute $E[Y]$ and σ_X carefully

Markov on $Y = (X - \mu_X)^2$

Markov

- $P\{Y \geq c\} \leq \frac{E[Y]}{c}$

- $p_X(k) = \begin{cases} 1/6 & 1 \leq k \leq 6 \\ 0 & else \end{cases}$

$P\{X \geq 4\} = \frac{E[X]}{4} = \frac{3.5}{4} = 0.875$

Chebyshev

- $P\{|X - \mu_X| \geq a\sigma_X\} \leq \frac{1}{a^2}$

$P\{X - 3.5 \geq 0.5\} \leq P\{|X - 3.5| \geq 0.5\}$

$0.5 = a\sigma_X$

PDF/ CDF

CDF

- Properties – None-decreasing/ output span [0,1]/ right cont.
- $F_X(c) = \int_{-\infty}^{c} f_x(u)du$– Remember the constant
- “Jump” at c ($F_X(c) > \mathrm{F_X(c-)}$) implies $p_X(c) > 0$

PDF

- $f_X = F_X{}'$
- $\int_{-\infty}^{\infty} f_X(u)du = 1$

$$P\{a < X \leq b\} = F_X(b) - F_X(a) = \int_a^b f_X(u)du$$

PDF/ CDF

$$f_X(x) = \begin{cases} c^2 e^{-5x} & x \geq 0 \\ 0 & else \end{cases}$$, find c, $F_X(x)$, $P\{1 \leq X \leq 3\}$

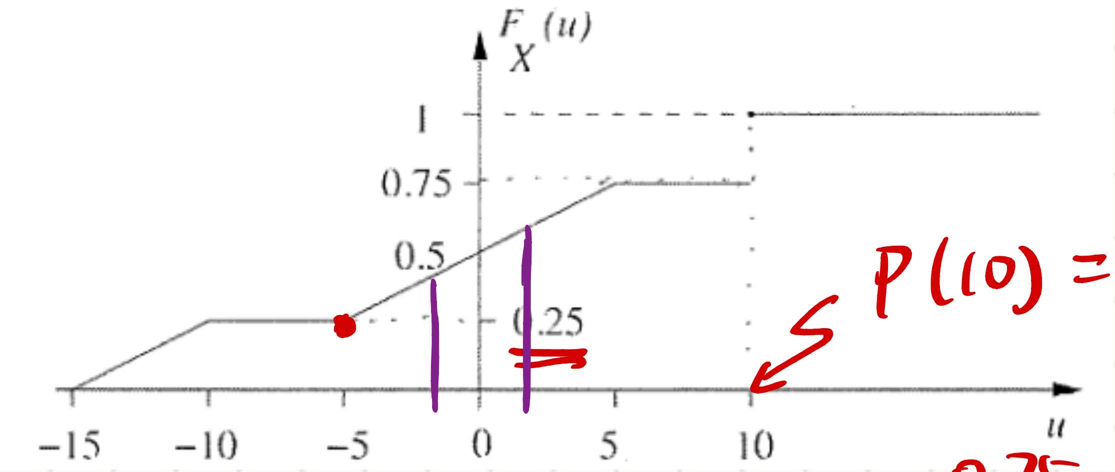
$$\int_0^\infty c^2 e^{-5x}\,dx = c^2\left(-\frac{e^{-5x}}{5}\right)\Big|_0^\infty = 1.$$

$$c^2 = 5 \qquad c = \pm\sqrt{5}.$$

$$f_X(x) = 5e^{-5x} = Exp(\lambda = 5)$$

$$F_X(x) = 1 - e^{-5x}. \qquad \int_1^3 5e^{-5x}\,dx = \_\_\_\_\_$$

PDF/ CDF



$P(10) = 0.25$

$f_X(u) = \frac{0.75-0.25}{5-(-5)} = \frac{0.5}{10} = 0.05$

- Solve $P\{X \leq -5\}$, $P\{X = 10\}$, $P\{X^2 \leq 4\}$, $E[X]$

$P\{X \leq -5\} = F_X(-5) = 0.25$

$P(X=10) = F_X(10) - F_X(10-) = 1 - 0.75$
$= 0.25$

$P\{X^2 \leq 4\} = \int_{-2}^{2} f_X(u)\,du = 4 \times 0.05 = 0.2$

Real-Time QA