

Caveats

- Problems in the slide will be independent from midterm problems
 - $P(p_1 | p'_1 \text{ in slide}) = P(p_1)$
- All numbers will be replaced by symbols in the slide
 - In midterm, you may need to compute
- We will cover top-K options from the Slido survey
 - Survey does not cover all topics
 - You still need to review all topics by yourself

Agenda & Survey result

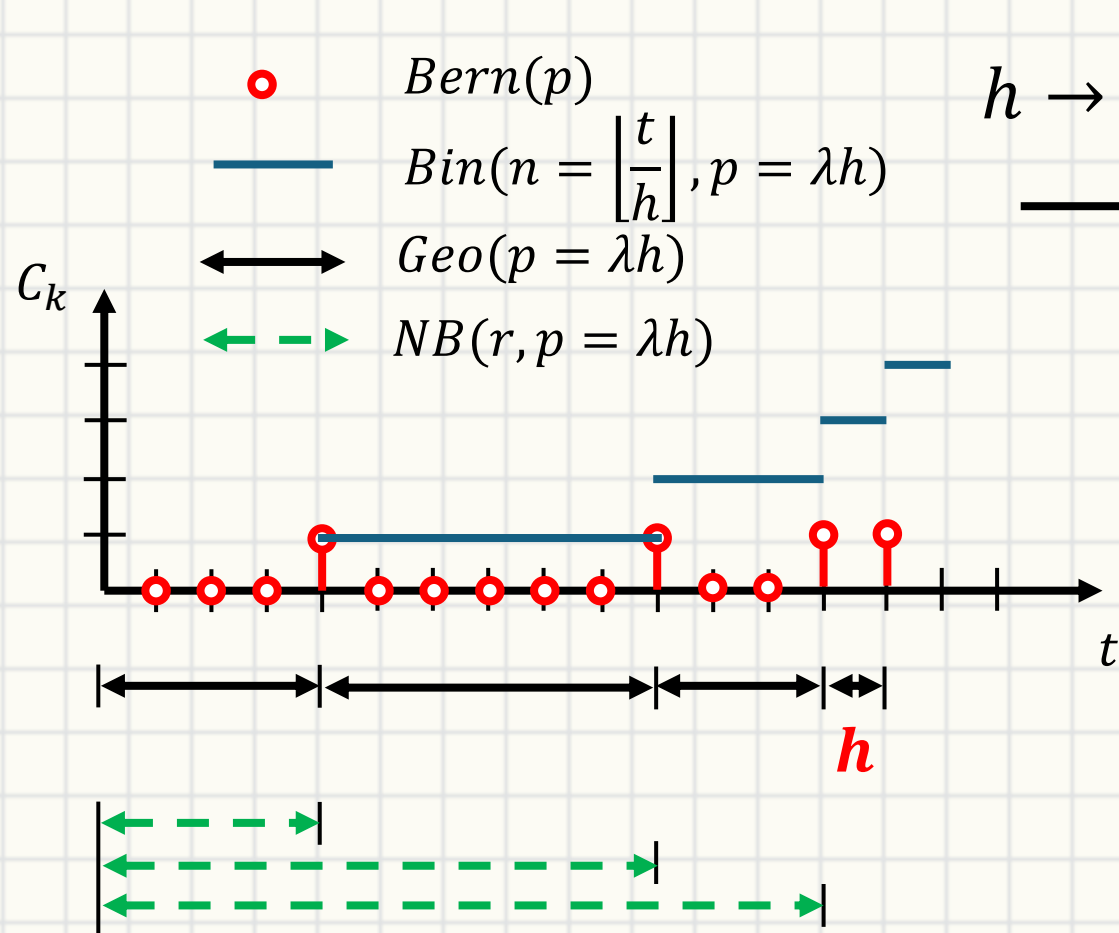
- Poisson Process
 - Exponential Distribution
- Scaling of PDF
- Markov and Chebyshev
- PDF/CDF

Bernoulli Process

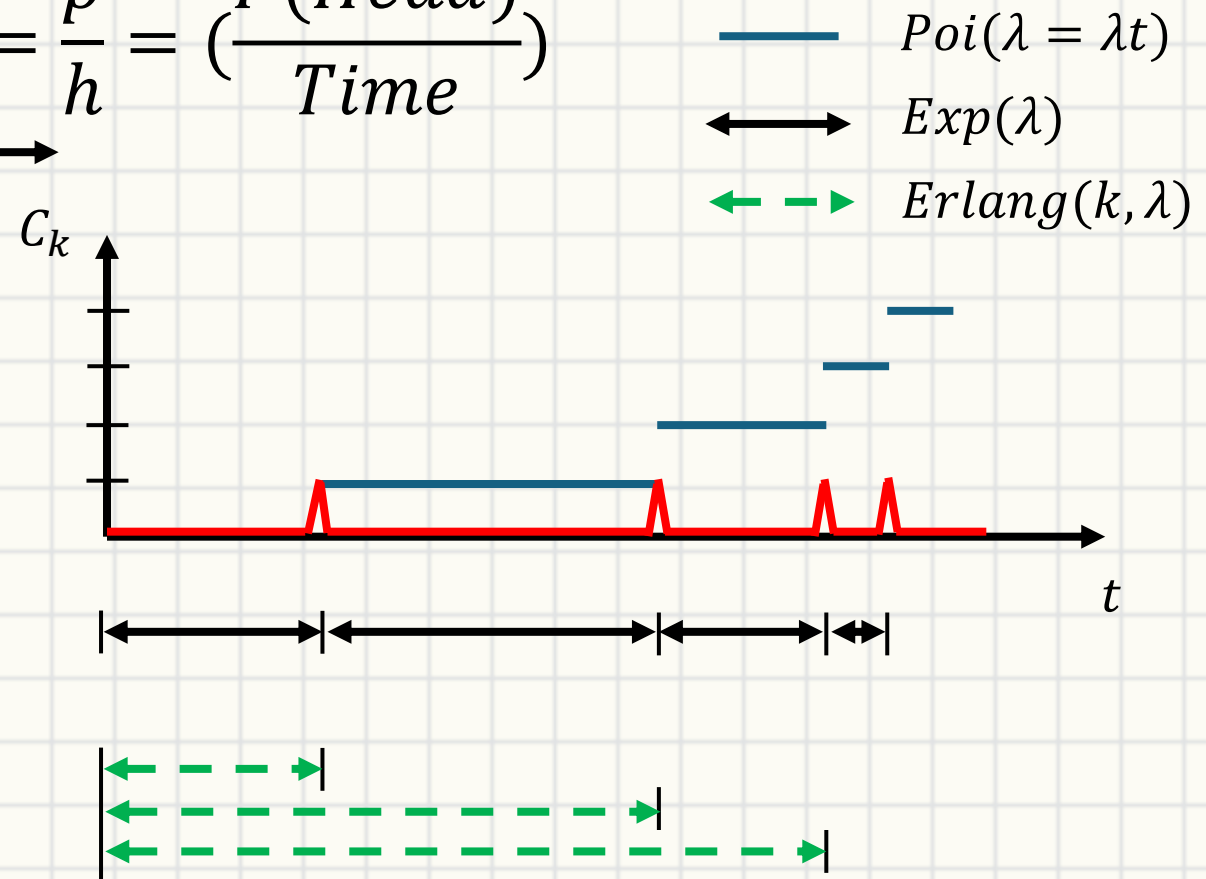
$$h \rightarrow 0, \lambda = \frac{p}{h}$$

Poisson Process

- Assume each trial takes h duration to complete



$$h \rightarrow 0, \lambda = \frac{p}{h} = \left(\frac{P(Head)}{Time} \right)$$



Properties

	$Exp(\lambda)$	$Poi(\lambda = \lambda_{ref}t)$
Mean		
Variance		
PDF/ PMF		
CDF		
Example	System lifetime	Event occurrence within t
Special		

Poisson Process

- $\lambda = \lambda_{ref} t$
- None-overlapped process are

A support center is receiving λ call/ mins. Probability of

- Exactly 4 calls in 2 mins
- At least 3 calls in 1 min
- 5 calls in 5 mins, given 2 calls in the third mins
- 5 calls in 5 mins, among which 2 calls in the third min

Scaling of PDF

Let $Y = aX + b$, where X, Y are RV and a, b are constants

- $f_Y(u) = f_X\left(\frac{u-b}{a}\right) \times \frac{1}{|a|}$

$$f_X(u) = \begin{cases} 1 & -0.5 \leq u \leq 0 \\ 1-u & 0 \leq u \leq 1 \end{cases}, \text{ let } Y = 2X + 1$$

- Solve $E[Y]$ and σ_Y^2

- $P\{Y \geq 2\}$

Markov and Chebyshev

- Check whether Y is none-negative
- Compute $E[Y]$ and σ_X carefully

Markov

- $P\{Y \geq c\} \leq \frac{E[Y]}{c}$

- $p_X(k) = \begin{cases} 1/6 & 1 \leq k \leq 6 \\ 0 & \text{else} \end{cases}$

Chebyshev

- $P\{|X - \mu_X| \geq a\sigma_X\} \leq \frac{1}{a^2}$

PDF/ CDF

CDF

- Properties – None-decreasing/ output span $[0,1]$ / right cont.
- $F_X(c) = \int_{-\infty}^c f_X(u)du$ – Remember the constant
- “Jump” at c ($F_X(c) > F_X(c-)$) implies $p_X(c) > 0$

PDF

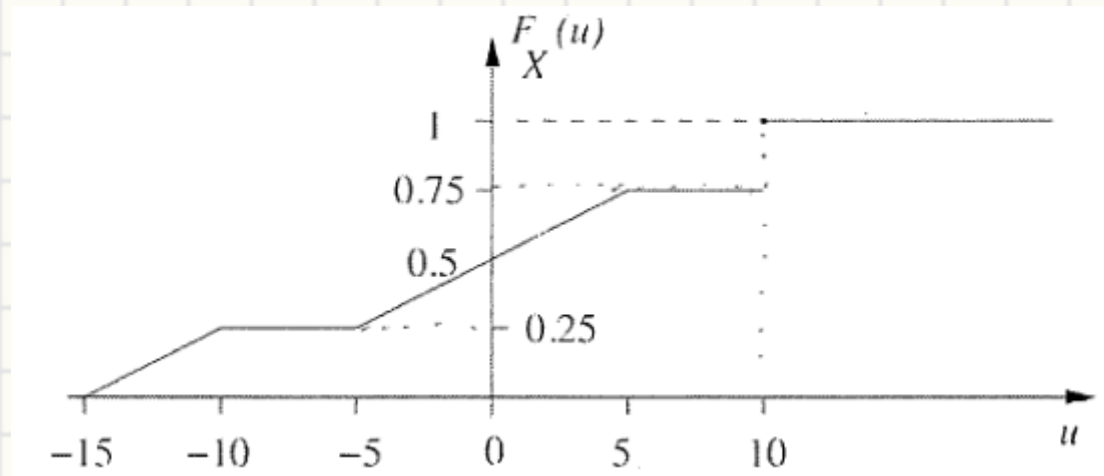
- $f_X = F_X'$
- $\int_{-\infty}^{\infty} f_X(u)du = 1$

$$P\{a < X \leq b\} = F_X(b) - F_X(a) = \int_a^b f_X(u)du$$

PDF/ CDF

$$f_X(x) = \begin{cases} c^2 e^{-5x} & x \geq 0 \\ 0 & \text{else} \end{cases}, \text{ find } c, F_X(x), P\{1 \leq X \leq 3\}$$

PDF/ CDF



- Solve $P\{X \leq -5\}$, $P\{X = 10\}$, $P\{X^2 \leq 4\}$, $E[X]$

Real-Time QA