Last lecture

Conditional Probability (Ch 2.3)

- Examples
- Solver
- 3 doors problem revisited

$P(B|A) =$

Law of Total Probability (Ch 2.10)

- Bayes formula

$P(A) =$

$P(E_i|A) =$

Agenda

Bayes formula (Ch 2.10)
- Examples

Independent Events/ RVs (Ch 2.4)
- Definition
- Motivation
- Examples and Facts

- Distributions (Ch 2.4)
  - Bernoulli
  - Binomial

Disease problems

Assume there is a disease A, and the corresponding test T

- What do the followings mean?
- $P(T|A) = 0.9$
- $P(T|A^c) = 0.05$
- $P(A) = 0.01$

- $P(A|T) =$

Disease problems

According to CDC survey on smoker

- 18% of adults are smokers
- 15% of women are smokers
- Population = 50% men + 50% women

- What fraction of adult smokers are women

Disease problems

According to CDC survey on smoker vs. lung cancer

- 15% of women are smokers
- Compared to nonsmokers, women who smoker are 13 times likely to get lung cancer

- If I pick a female lung cancer patient, how likely she is a smoker?

Independent Events/ RVs

Definition

A and B are events, they are if

- $P(B|A) = P(B)$ or
- $P(AB) = P(A)P(B)$

Facts

- $P(B|A) = P(B)$ implies $P(A|B) = P(A)$
- If $P(A) = 0$, B is independent of A

Definition

A and B are events, they are if

- $P(B|A) = P(B)$ or
- $P(AB) = P(A)P(B)$

RVs X and Y are independent if A and B are independent for any $X \in A$ and $Y \in B$

Motivation

Independent is a common but strong property

- $P(AB) = P(A)P(B)$ the pmf
  - Compute the pmf easily
  - Will skip Ch 2.3 affect only my HW2 and Midterm 1?
  - If I join this club, will it affect my GPA?

- Decide the model complexity
  - What really affects the results?
  - What do I need to ask when reviewing a loan request?
  - What input data do I need to predict the defect?

Examples

Physically independence – Toss a coin and roll a die (N, X)

- $A \triangleq \{N = H\}$
- $B \triangleq \{X = 6\}$

Probabilistic independence

- $A \triangleq X$ is even
- $B \triangleq \{X \equiv 0\ (mod\ 3)\}$

Examples

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Terms and Facts

- A, B, C are pairwise independent if $(A, B), (B, C), (A, C)$ are mutually independent
  - Toss a fair coin twice
  - $A \triangleq$ {First coin is Head}
  - $B \triangleq$ {second coin is Head}
  - $C \triangleq$ {toss results are the same}

Terms and Facts

- A, B, C are independent they are pairwise independent and $P(ABC) = P(A)P(B)P(C)$

- $A_1, A_2, \ldots A_i$ are independent if

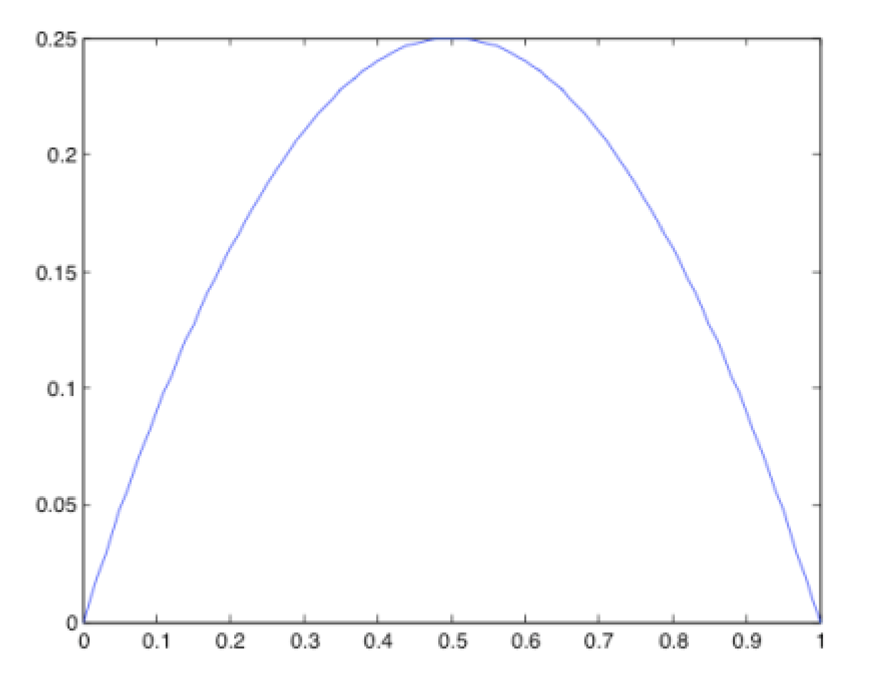
$$P\left(A_{i_1} A_{i_2} \ldots A_{i_k}\right) = P\left(A_{i_1}\right) P\left(A_{i_2}\right) \ldots P(A_{i_k})$$

Distributions

Bernoulli Distribution

X is Bernoulli distribution with parameter p if

- $P\{X = 1\} = p$ and $P\{X = 0\} = 1 - p$
- "Toss a (unfair) coin with p probability Head"
- Only possible outcomes, pmf contains bins
- $E[X] =$
- $E[X^2] =$
- $\sigma_x^2 =$



Binomial Distribution

X is binomial distribution with parameter (n, p) if

- X is sum of n Bernoulli trials with parameter p
- Draw the unfair coin n times and count the Head

- $p_X(k) =$