Last lecture

Random Variables (RV)

- Variance (Ch 2.2)

Conditional Probability (Ch 2.3)

- Motivation
- Examples

Agenda

Conditional Probability (Ch 2.3)

- Examples
- Solver
- 3 doors problem revisited

Law of Total Probability (Ch 2.10)

- Bayes formula

Conditional Probability

Conditional Probability

$$P(B|A) = \begin{cases} \dfrac{P(A,B)}{P(A)} & \text{If } P(A) > 0 \\ \text{Undefined} & \text{Else} \end{cases}$$

Roll two dice, $A =$ sum is 6; $B =$ numbers are not equal

$P(B) =?$ $P(B|A) =?$ $P(B^c|A) =?$

Conditional Probability

$$P(A,B) = P(B|A) \times P(A) \qquad \text{If } P(A) > 0$$

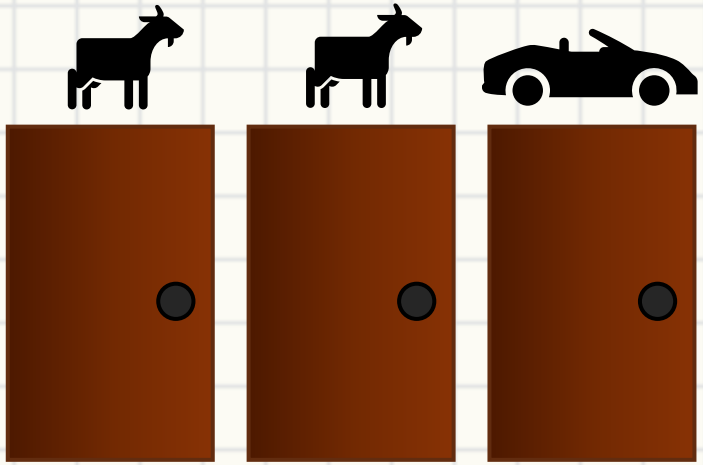
In many cases, we might only know some probabilities…

- 3 doors problem A: $x_1 = Car$, if we change…
  - $P(W|A)$
  - $P(W,A)$
  - $P(W,A^c)$

Facts of conditional probability

- $P(B|A) > 0$
- $P(B|A) + P(B^c|A) = 1$
- $P(\Omega|A) = 1$
- $P(AB) = P(A|B)P(B)$
- $P(ABC) = P(A|BC)P(B|C)P(C)$

Examples



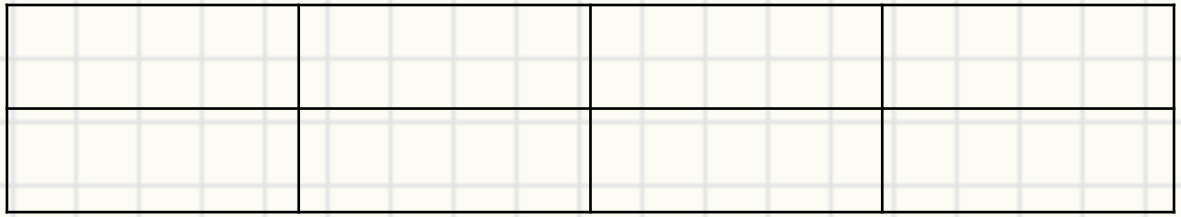
- Never change
  - $P(W|X_1 = C) =$
  - $P(W|X_1 = G) =$

- Change
  - $P(W|X_1 = C) =$
  - $P(W|X_1 = G) =$

- What if there are 4 doors… 2 cars and 2 goats?

Law of total probability

Law of total probability



A

A^c

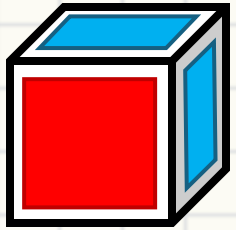
- Case-by-case discussion law…
- $P(A)$ is the summed of “Partitioned conditional probability”

- $P(A) = \sum_i P(A|E_i)P(E_i)$

Law of total probability

There are 3 dice A, B, C in the bag

- $A = [R \times 1; B \times 5]$
- $B = [R \times 2; B \times 4]$
- $C = [R \times 3; B \times 3]$



Draw one die and roll many times

- $P(R_1)$

- $P(R_2|R_1)$

Bayes Formula

Conditional probability + Law of total probability

- How do we get $P(B|A)$ from $P(A|B)$?

- $P(B|A) =$

- $P(E_i|A) =$

Disease problems

Assume there is a disease A, and the corresponding test T

- What do the followings mean?
- $P(T|A) = 0.9$
- $P(T|A^c) = 0.05$
- $P(A) = 0.01$

- $P(A|T) =$

Disease problems

According to CDC survey on smoker

- 18% of adults are smokers
- 15% of women are smokers
- Population = 50% men + 50% women

- What fraction of adult smokers are women

Disease problems

According to CDC survey on smoker vs. lung cancer

- 15% of women are smokers
- Compared to nonsmokers, women who smoker are 13 times likely to get lung cancer

- If I pick a female lung cancer patient, how likely she is a smoker?