# Last lecture

Random Variables (RV)

- Probability Mass Function (pmf)
- Mean and Variance (Ch 2.2)

# Agenda

Random Variables (RV)

- Mean and Variance (Ch 2.2)

Conditional Probability (Ch 2.3)

- Motivation
- Examples
- Solver

Law of Total Probability (Ch 2.10)

# Variance and Standard Deviation

Mean is important… but not complete enough

- Variance $Var(X)$ is how PMF spreads apart from $\mu_X$

- $Var(X) \triangleq E[(X-\mu_x)^2] = E[X^2] - (E[X])^2$

# Variance and Standard Deviation

Standard deviation $\sigma_X \triangleq \sqrt{Var(X)}$ ; $Var(X) = \sigma_X^2$

$\sigma_X$ is of the same unit as $X$

$Var(X + c) =$

$Var(aX + c) =$

$2X$ vs $X + X$

# Standardized RV

For any RV $X$

- $\frac{X-\mu_X}{\sigma_X}$ is a RV – Mean 0, variance 1

# Conditional Probability

# Motivation

The probability of $B$ happens given $A$ happens

- $P($ pair of socks are same color $)$ given $S_1 = B$
- $P(I\ win\ Texas\ Hold'em)$ given $X = Ace + Ace$
- $P(I\ pass\ 313)$ given I skip HW1…

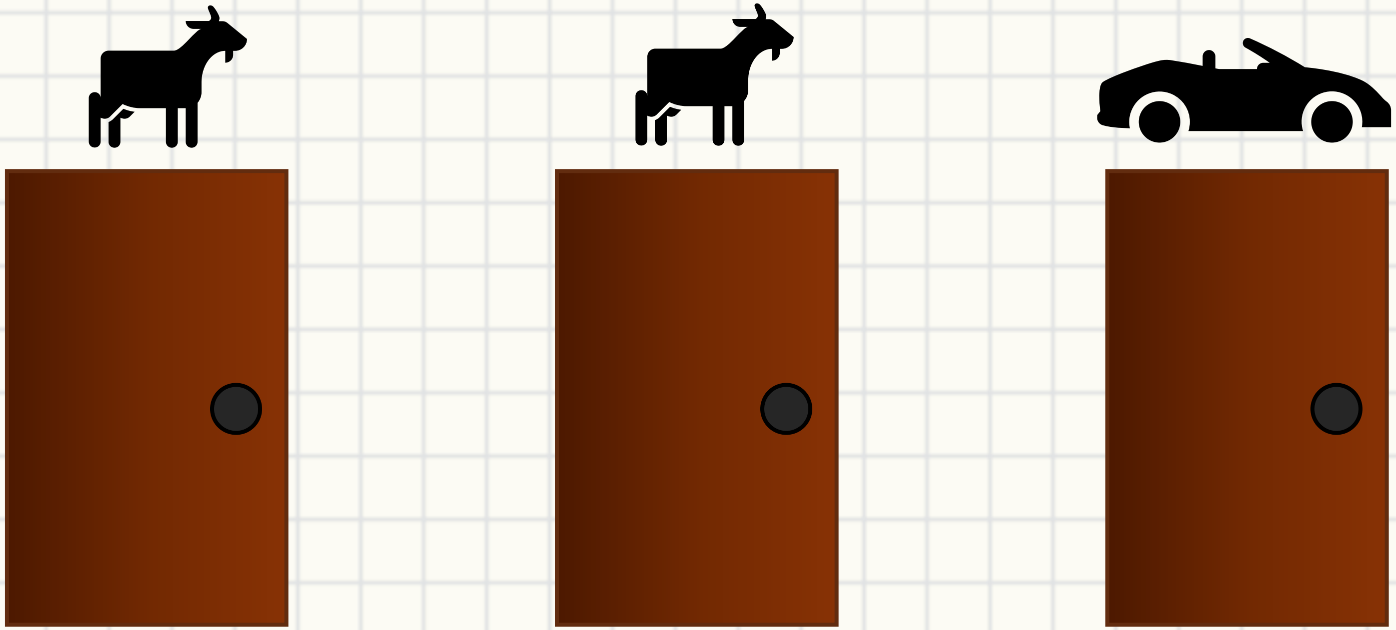
Why do we need conditional probability?

- Analyze the relationship between two events
- Find the optimal solution to make an event probable

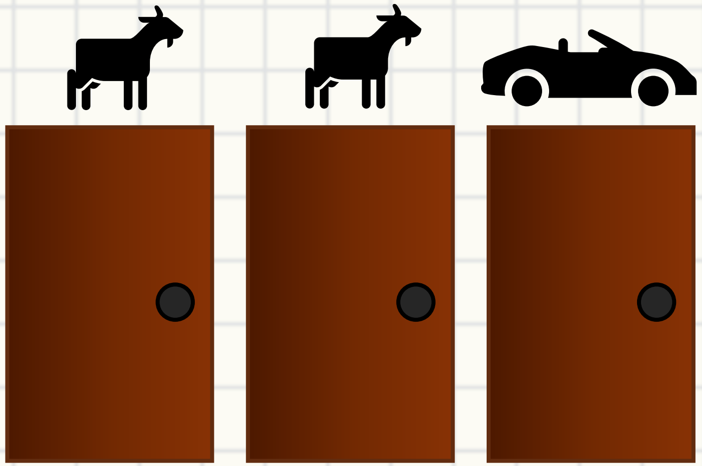
# Examples

3 doors problem

- 3 closed doors – 1 leads to a car, the others lead to goats
- After you choose one, the host will open a "Fail" door
- Should you change the door?



# Examples



- Never change
  - $P(W|X_1 = C) =$
  - $P(W|X_1 = G) =$

- Change
  - $P(W|X_1 = C) =$
  - $P(W|X_1 = G) =$

- What if there are 4 doors… 2 cars and 2 goats?

# Conditional Probability

$$P(B|A) = \begin{cases} \\ \end{cases}$$

Roll two dice, $A =$ sum is 6; $B =$ numbers are not equal

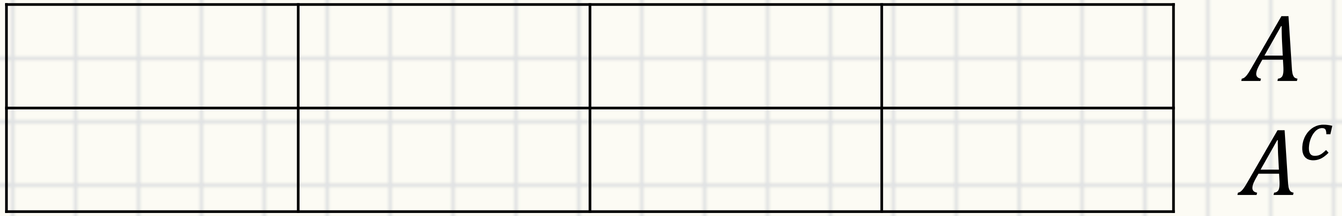
$P(B) =?$ $P(B|A) =?$ $P(B^c|A) =?$

# Facts of conditional probability

- $P(B|A) > 0$
- $P(B|A) + P(B^c|A) = 1$
- $P(\Omega|A) = 1$
- $P(AB) = P(A|B)P(B)$
- $P(ABC) = P(A|BC)P(B|C)P(C)$

# Law of total probability

# Law of total probability



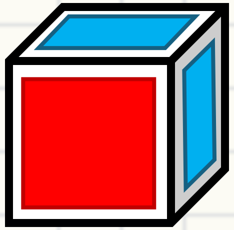

- Case-by-case discussion law…
- $P(A)$ is the summed of “Partitioned conditional probability”

- $P(A) = \sum_i P(A|E_i)P(E_i)$

# Law of total probability

There are 3 dice $A, B, C$ in the bag

- $A = [R \times 1; B \times 5]$
- $B = [R \times 2; B \times 4]$
- $C = [R \times 3; B \times 3]$



Draw one die and roll many times

- $P(R_1)$

- $P(R_2|R_1)$