Last lecture

Probability axioms(Ch 1.2 cont.)

- Karnaugh map (cont.)

- Axioms of Probability

Agenda

Counting the size of events (Ch 1.3)

- Independent events
- Dependent but countable

Probability with equally likely outcomes (Ch 1.4)

- Draw socks from the drawer
- Poker hands

Counting the size of events

How large is A and Ω

- If events contain outcomes
  - $P(A) =$

- But how large is $||A||$ and $||\Omega||$?
- Independent experiments
  - Toss a coin and roll a die
  - Roll a die twice
  - Draw 3 balls out of 5 balls replacement
- Dependent
  - Bin of balls $\Omega = \{\text{Red}, \text{Red}, \text{Red}, \text{Green}, \text{Green}\}$
  - Draw two balls
  - Pokers

Independent experiments

- If we toss a coin and roll a die
- $\Omega_c =$ $\Omega_d =$
- $\Omega =$
- $\left||\Omega|\right| =$
- $\left||A|\right| =$
- Independent events $P(AB) =$
- Principle of counting:

Dependent experiments

- What if the first draw affects the second one?
- Example:
  - I have 4 pairs of black socks and 2 pairs of white socks
  - $P(Draw\ two\ socks, color\ is\ the\ same)$?

  - $\Omega_1 = \{ \quad \}$
  - $A = \{ \quad \}$

- We need a tool – &

Permutation

- The   to order n different items
- How many ways can you order letters A, B, C, D?

- N letters ->
- What if I want to order "A, B, C ... G “ 7 letters, but only pick the first 4?

- What if the order doesn’t matter?

Principle of over-counting

- What if I want to order letters ILLINI?

- For an integer $K \geq 1$, if each element of a set is counted K times, then the number of elements in the set is the total count divided by K

Combination

- $\binom{n}{k}$ or $C(n,k)$
  - The   to choose k out of n different items
  - $\binom{n}{k} =$

  - Draw 3 balls out of 5 balls   replacement

The Socks Problem

I have 4 pairs of black socks and 2 pairs of white socks

$P(Draw\ two\ socks, color\ is\ the\ same)$?

Poker Problem

$$\Omega_{card} =$$

Draw 5 cards out of 52 cards

FULL HOUSE = 3 same numbers, other 2 same numbers

$$P(FULL\ HOUSE) =$$

Sample space with infinite cardinality

Interval probability space

- $\Omega = \{\omega\colon 0 \leq \omega \leq 1\}$ $P([a, b]) = b - a$
- $A = [0.2, 0.8]$

And others…