

Last lecture

The Central Limit Theorem and Gaussian Approximation ([Ch 3.6.3](#))

- Examples

ML estimation for continuous RVs ([Ch 3.7](#))

- Definition
- Examples

Functions of a random variable ([Ch 3.8](#))

- Find CDF/ PDF of $g(X)$ ([Ch 3.8.1](#))

Agenda

Functions of a random variable (Ch 3.8)

- Find CDF/ PDF of $g(X)$ (Ch 3.8.1)
- Examples for
 - General
 - X is Gaussian
 - Case by case
 - g is cosine/ tangent
 - g is strictly increasing

Find CDF/ PDF of $g(X)$

Motivation – I know X follows some distribution

- but what about $Y = g(X)$?
1. Scope the problem - Find **support** of X and Y , are they continuous or discrete?
 2. Find $F_Y(c)$ from integrating $f_X(x)$ over $\{x: g(x) \leq c\}$
 - If Y is discrete, normally we can find pmf $p_Y(c)$
 3. Get $f_Y = F_Y'$

Examples - General

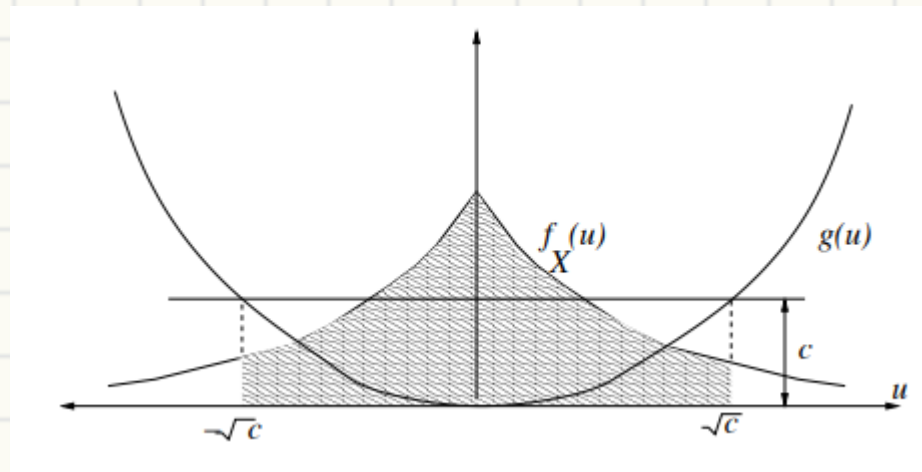
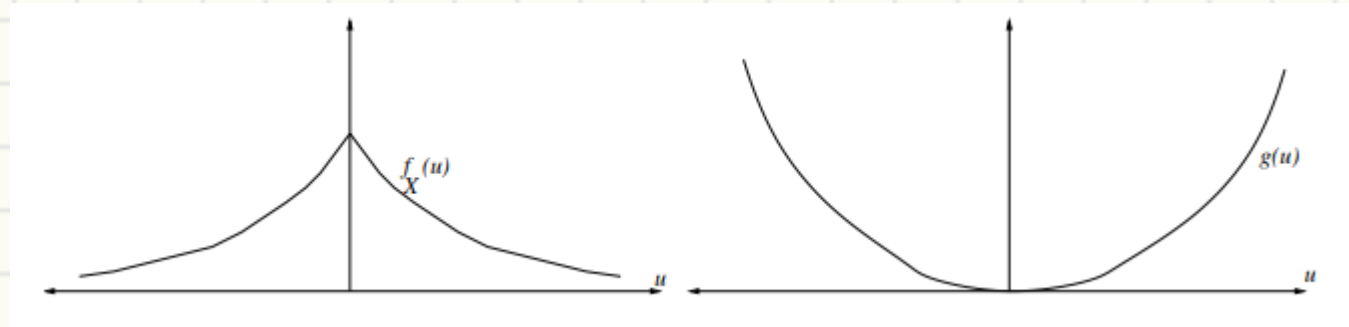
1. Find support and continuity
2. $F_Y(c) = \int_{x:f(x) \leq c} f_X(x) dx$
3. $f_Y = F_Y'$

RV X follows $f_X(u) = \frac{e^{-|u|}}{2}$ for $u \in \mathbb{R}$. $Y = X^2$. Find f_Y , μ_Y and σ_Y^2

1. $F_Y =$

2. $P\{X^2 \leq c\} =$

3. $f_Y(c) =$



Examples - Gaussian

1. Find support and continuity
2. $F_Y(c) = \int_{x:f(x)\leq c} f_X(x)dx$
3. $f_Y = F_Y'$

Let $Y = X^2$, where $X \sim N(\mu = 2, \sigma^2 = 3)$, find f_Y

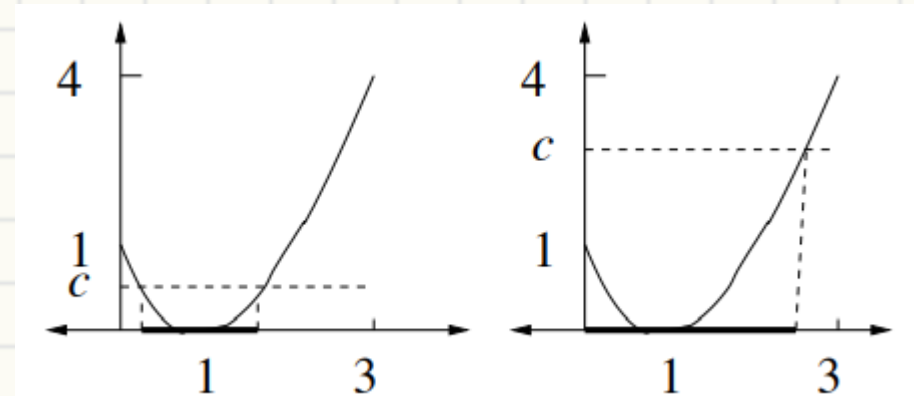
- $F_Y(c) =$
- $f_Y(c) = F_Y'(c)$

Examples – Case by case

1. Find support and continuity
2. $F_Y(c) = \int_{x:f(x) \leq c} f_X(x) dx$
3. $f_Y = F_Y'$

Let $Y = (X - 1)^2$, where $X \sim \text{Uni}(0,3)$, find f_Y

- $F_Y(c)$
 - $0 \leq c \leq 1$
 - $1 \leq c \leq 4$
- $f_Y(c) = F_Y'(c)$



Midterm Review Poll

- Halloween (Oct.31) class time
- Vote for the contents!
- Optional questions on general classroom feedback



#3976074

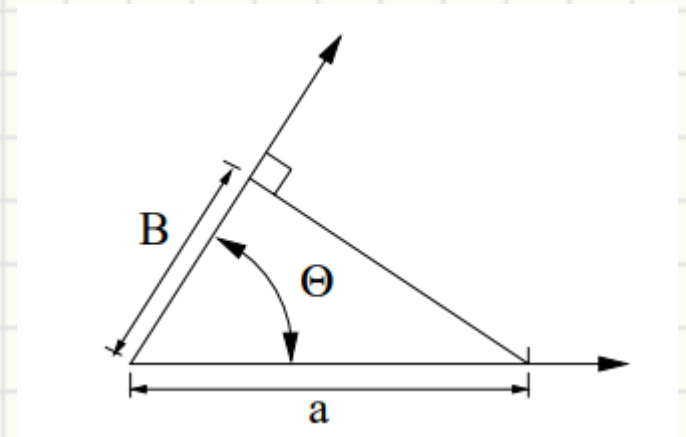
Examples – Cosine

1. Find support and continuity
2. $F_Y(c) = \int_{x:f(x) \leq c} f_X(x) dx$
3. $f_Y = F_Y'$

Let $B = a \cos \Theta$, where $\Theta \sim \text{Uni}(-\pi, \pi)$, find f_B

- Useful in DSP
- $F_B(c)$

- $f_B(c) = F_B'(c)$
 - Hint $\frac{d(\cos^{-1} x)}{dx} = -(1 - x^2)^{-\frac{1}{2}}$

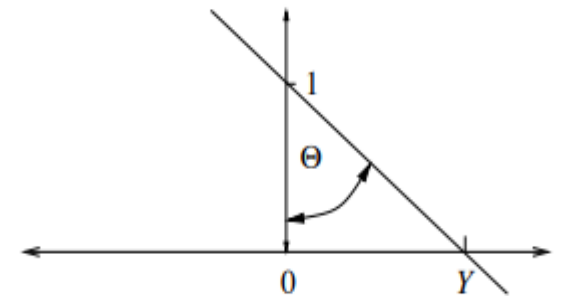


Examples – Tangent

1. Find support and continuity
2. $F_Y(c) = \int_{x:f(x) \leq c} f_X(x) dx$
3. $f_Y = F_Y'$

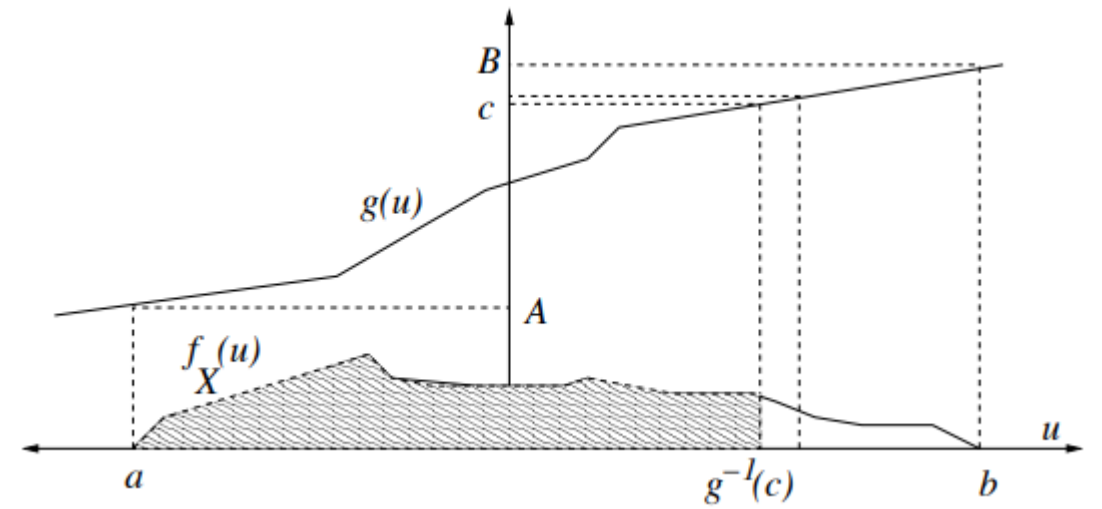
Let $Y = \tan \Theta$, where $\Theta \sim \text{Uni} \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$, find f_Y

- $F_Y(c)$
- $f_Y(c) = F_Y'(c)$



Increasing g function

- X has support $[a, b]$
- $Y = g(X)$
- g is strictly increasing
- Y has support $[g(a), g(b)] = [A, B]$
- Find $F_Y(c)$ where $A \leq c \leq B$



- $f_Y(c) =$

Creating uniform distribution

- Let $Y = F_X(X)$
- What is F_Y ?