Last lecture

Gaussian (normal) Distribution (Ch 3.6.2)

- Example

The Central Limit Theorem and Gaussian Approximation (Ch 3.6.3)

- Definition
- CDF Approximation
- Examples

Agenda

The Central Limit Theorem and Gaussian Approximation (Ch 3.6.3)

- Examples

ML estimation for continuous RVs (Ch 3.7)

- Definition
- Examples

Functions of a random variable (Ch 3.8)

- Find CDF/ PDF of g(X) (Ch 3.8.1)

Example

$X \sim Bin(n = 1000, p = 0.5)$, Using Gaussian approximation, find K s.t. $P\{X \geq K\} \approx 0.01 = \mathrm{Q}(2.325)$

- $\mu_X =$ $\quad\quad\quad\quad$ $, \sigma_X = \sqrt{np(1-p)} = \sqrt{250} \approx 15.8$
- $P\{X \geq K\} =$

- What if $n = 1000000$?

Example

We want to estimate p with $\hat{p} = \frac{X}{n}$

- Find $P\{|\hat{p} - p| < \delta\}$ in terms of n, p, δ, and Φ

- Find δ w/ 99% confidence if $p = 0.5$, $n = 1000$. Given that $\Phi(2.58) \approx 0.995$

- What if $p = 0.1$?

ML estimation

Definition

Recall for discrete RV X, given observation u, ML is to find θ maximizing $p_\theta(u)$

- But for continuous RV, $p_\theta(u) = 0$
- Instead, ML maximize $f_\theta(u)$ because
  - $f_\theta(u) \approx \frac{1}{\epsilon} P\{u - \frac{\epsilon}{2} < X < u + \frac{\epsilon}{2}\}$
  - $\hat{\theta}_{ML}(u) \triangleq argmax_\theta\ f_\theta(u)$

Example

$T \sim Exp(\lambda)$ where λ is unknown. We want to estimate λ with observation t

- $f_T(t) =$

- Extrema happens at $\frac{df_T(t)}{d\lambda} = 0$

Example

$X \sim Uni([0, b])$ where b is unknown. We want to estimate b with observation u

- $f_X(u) =$

- Extrema happens at

Functions of a random variable

Find CDF/ PDF of g(X)

Motivation – I know X follows some distribution

- but what about $Y = g(X)$?

1. Scope the problem - Find of X and Y, are they continuous or discrete?
2. Find $F_Y(c)$ from integrating $f_X(x)$ over $\{x: g(x) \leq c\}$
   - If Y is discrete, normally we can find pmf $p_Y(c)$
3. Get $f_Y = F_Y{}'$

Examples

1. Find support and continuity
2. $F_Y(c) = \int_{x:f(x)\le c} f_X(x)dx$
3. $f_Y = F_Y{}'$

RV X follows $f_X(u) = \frac{e^{-|u|}}{2}$ for $u \in R$. $Y = X^2$. Find f_Y, μ_Y and σ_Y^2

1. $F_Y =$

2. $P\{X \le c\} =$

3. $f_Y(c) =$