Last lecture

Gaussian (normal) Distribution (Ch 3.6.2)

Example

The Central Limit Theorem and Gaussian Approximation (Ch 3.6.3)

- Definition
- CDF Approximation
- Examples

Agenda

The Central Limit Theorem and Gaussian Approximation (Ch 3.6.3)

Examples

ML estimation for continuous RVs (Ch 3.7)

- Definition
- Examples

Functions of a random variable (Ch 3.8)

Find CDF/ PDF of g(X) (Ch 3.8.1)

 $X \sim Bin(n = 1000, p = 0.5)$, Using Gaussian approximation, find K s.t. $P\{X \ge K\} \approx 0.01 = Q(2.325)$

•
$$\mu_X = ,\sigma_X = \sqrt{np(1-p)} = \sqrt{250} \approx 15.8$$

- $P\{X \geq K\} =$
- What if n = 1000000?

We want to estimate p with $\hat{p} = \frac{X}{n}$

• Find $P\{|\hat{p}-p|<\delta\}$ in terms of n,p,δ , and Φ

- Find δ w/ 99% confidence if p=0.5, n=1000. Given that $\Phi(2.58)\approx 0.995$
- What if p = 0.1?



Definition

Recall for discrete RV X, given observation u, ML is to find θ maximizing $p_{\theta}(u)$

- But for continuous RV, $p_{\theta}(u) = 0$
- Instead, ML maximize $f_{\theta}(u)$ because
 - $f_{\theta}(u) \approx \frac{1}{\epsilon} P\{u \frac{\epsilon}{2} < X < u + \frac{\epsilon}{2}\}$ $\hat{\theta}_{ML}(u) \triangleq argmax_{\theta} f_{\theta}(u)$

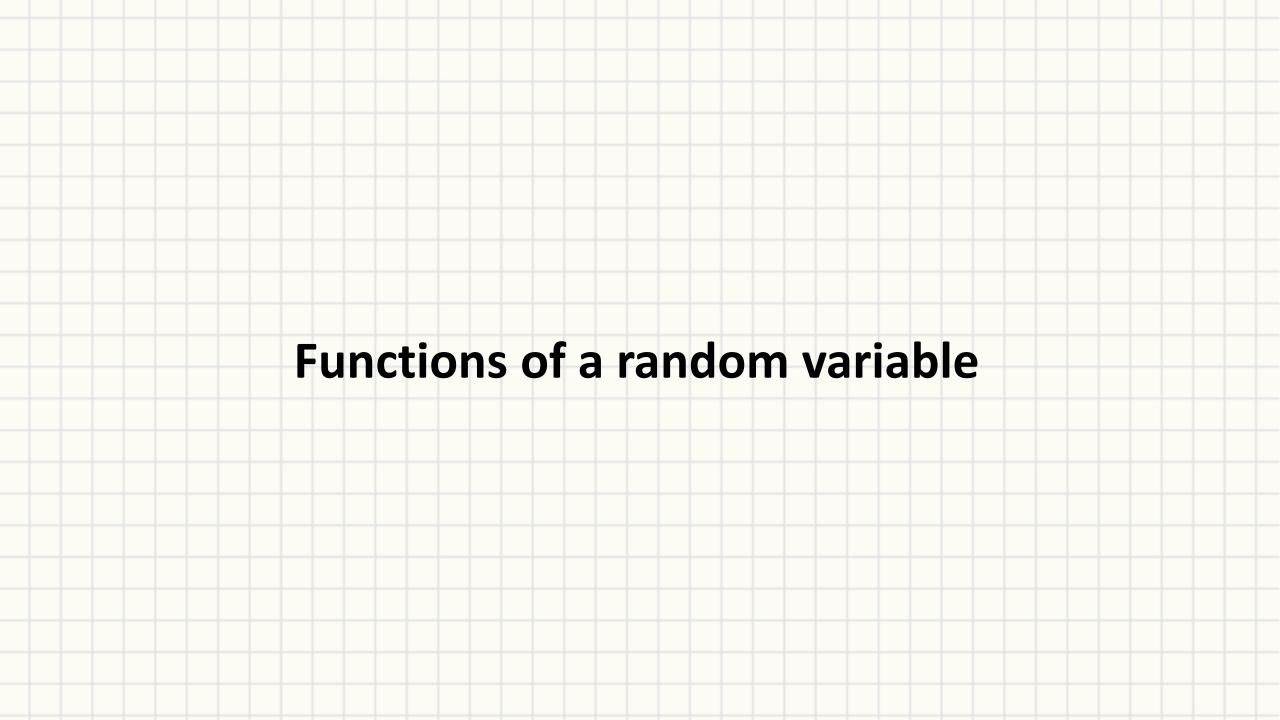
 $T \sim Exp(\lambda)$ where λ is unknown. We want to estimate λ with observation t

•
$$f_T(t) =$$

• Extrema happens at $\frac{df_T(t)}{d\lambda} = 0$

 $X \sim Uni([0,b])$ where b is unknown. We want to estimate b with observation u

- $f_X(u) =$
- Extrema happens at



Find CDF/ PDF of g(X)

Motivation – I know X follows some distribution

- but what about Y = g(X)?
- 1. Scope the problem Find continuous or discrete?

of X and Y, are they

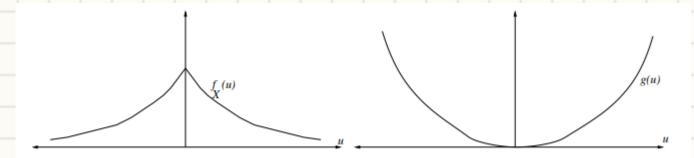
- 2. Find $F_Y(c)$ from integrating $f_X(x)$ over $\{x: g(x) \le c\}$
 - If Y is discrete, normally we can find pmf $p_Y(c)$
- 3. Get $f_Y = F_Y'$

- 1. Find support and continuity
- 2. $F_Y(c) = \int_{x:f(x) \le c} f_X(x) dx$ 3. $f_Y = F_Y'$

RV X follows $f_X(u) = \frac{e^{-|u|}}{2}$ for $u \in R$. $Y = X^2$. Find f_Y , μ_Y and σ_Y^2

1.
$$F_Y =$$

2.
$$P\{X \le c\} =$$



3. $f_Y(c) =$

