Last lecture

Gaussian (normal) Distribution (Ch 3.6.2)

- Motivation and Definition
- Examples

The Central Limit Theorem and Gaussian Approximation (Ch 3.6.3)

- Definition
- CDF Approximation
- Examples

Agenda

Gaussian (normal) Distribution (Ch 3.6.2)

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The Central Limit Theorem and Gaussian Approximation (Ch 3.6.3)

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ML estimation for continuous RVs (Ch 3.7)

- Definition
- Examples

Examples

Suppose $\mu_X = 10$ and $\sigma_X^2 = 3$. Compute $P\{X < 10 - \sqrt{3}\}$ if

- X is a Gaussian RV in terms of Q
- X is a uniform RV

(Hint: $10 - \sqrt{3} \approx 8.27$)

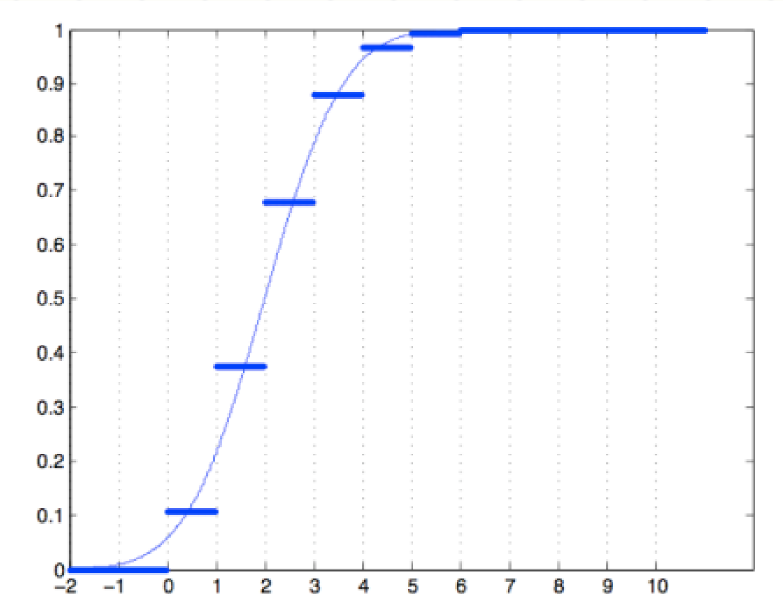
Central Limit Theorem and Gaussian Approximation

Central Limit Theorem (CLT)

If many independent random variables are added together, and if each of them is small in magnitude compared to the sum, then the sum X has an approximately Gaussian distribution $\tilde{X}$.

- $P\{X \le v\} \approx P\{\tilde{X} \le v\}$
- E.g., $X \sim Binomial(n, p)$ when np and $n(1-p)$ are not small
    - $(n, p) = (10, 0.2)$
    - $\mu_X =$
    - $\sigma_X^2 =$

    - What if np is small?

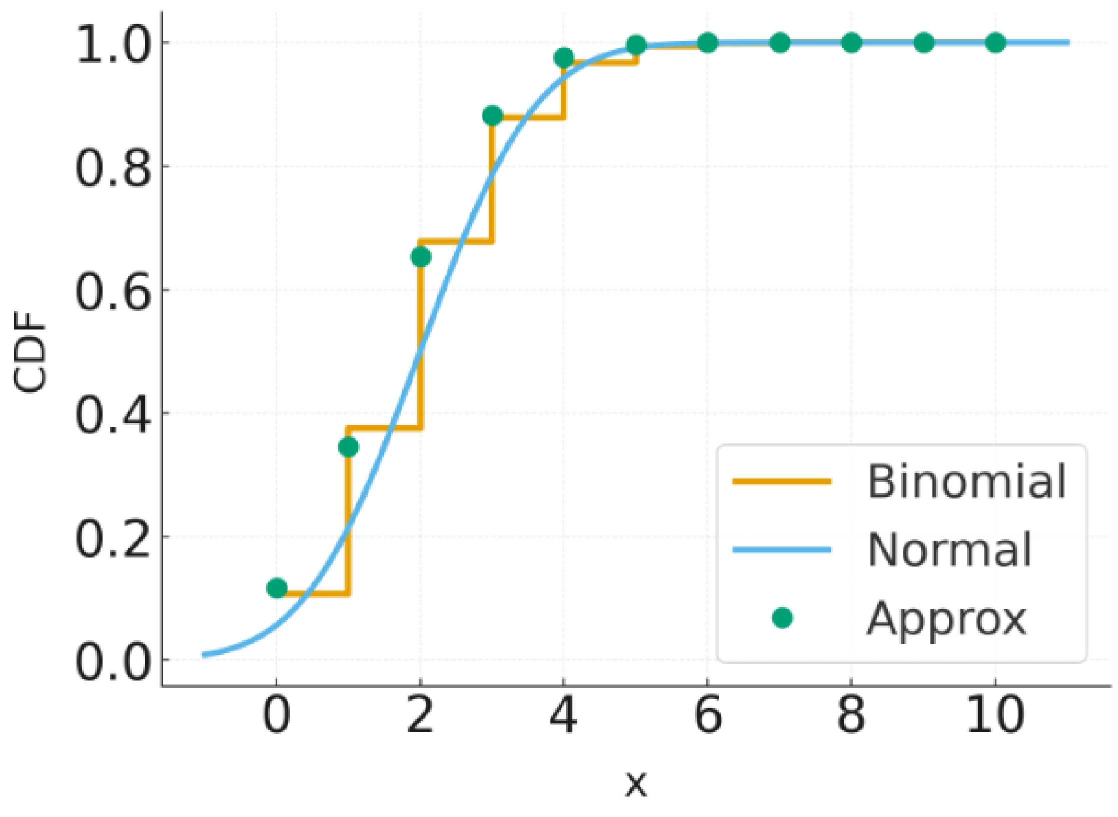


Gaussian Approximation

Approximate $X \sim Bin(10, 0.2)$ with $\tilde{X} \sim N(2, 1.6)$

- $F_X(2) = F_X(2.1) = F_X(2.9)$
- $F_{\tilde{X}}(2) \neq F_{\tilde{X}}(2.9)$
- How should we approximate?

- $P\{X \leq k\} \approx$
- $P\{X < k\} \approx$
- $P\{X \geq k\} \approx$
- $P\{X > k\} \approx$



Standardized Binomials

Standardized $S_{n,p} \sim Bin(n,p)$

- $X = \frac{S_{n,p} - np}{\sqrt{np(1-p)}}$

DeMoivre-Laplace limit theorem (First CLT version)

$$\lim_{n\to\infty} P\left\{\frac{S_{n,p} - np}{\sqrt{np(1-p)}} \leq c\right\} = \Phi(c)$$

Example

$X \sim Bin(n = 1000, p = 0.5)$, Using Gaussian approximation, find K s.t. $P\{X \geq K\} \approx 0.01 = \mathrm{Q}(2.325)$

- $\mu_X =$, $\sigma_X = \sqrt{np(1-p)} = \sqrt{250} \approx 15.8$
- $P\{X \geq K\} =$

- What if $n = 1000000$?

Example

We want to estimate p with $\hat{p} = \frac{X}{n}$

- Find $P\{|\hat{p} - p| < \delta\}$ in terms of n, p, δ, and Φ

- Find δ w/ 99% confidence if $p = 0.5$. Given that $\Phi(2.58) \approx 0.995$

- What if $p = 0.1$?

ML estimation

Definition

Recall for discrete RV X, given observation u, ML is to find θ maximizing $p_\theta(u)$

- But for continuous RV, $p_\theta(u) = 0$
- Instead, ML maximize $f_\theta(u)$ because
  - $f_\theta(u) \approx \frac{1}{\epsilon} P\{u - \frac{\epsilon}{2} < X < u + \frac{\epsilon}{2}\}$
  - $\hat{\theta}_{ML}(u) \triangleq argmax_\theta \; f_\theta(u)$

Example

$T \sim Exp(\lambda)$ where λ is unknown. We want to estimate λ with observation t

- $f_T(t) =$
- Extrema happens at $\frac{df_T(t)}{d\lambda} = 0$