Last lecture

Poisson process (Ch 3.5)

- Motivation
- Bernoulli process to Poisson process
- Definition
- Properties

Agenda

Poisson process (Ch 3.5) Examples

Erlang Distribution (Ch 3.5.3) Definition

Linear Scaling (Ch 3.6.1)

- Equation and derivation
- Examples

Gaussian (normal) Distribution (Ch 3.6.2)

- Motivation and Definition

Example

Calls arrive to a support center at rate $\lambda = 2$ calls per minute.
Let N_t denotes the number of calls until time t (mins)

- $N_t \sim$
- $P_{N_t}(k) =$
- $P\{$ No calls arrive in the first 3.5 minutes $\}$
- $P\{$ The third call arrives after time t = 3.$\}$

Erlang Distribution

Definition

Let T_r denotes the time of r^{th} count of a Poisson process

- $T_r = \sum_{i=1}^{r} U_i\ , U_i \sim Exp(\lambda)$
- $F_{T_r}^{c}(\mathrm{t}) = \mathrm{P}\{\mathrm{T_r} > \mathrm{t}\}$: “At most $r-1$ count by time t”
- $F_{T_r}^{c}(\mathrm{t}) =$
- $f_{T_r}(t) = -\frac{dF_{T_r}(t)}{dt} =$
- Intuitively – Exactly arrival before time t, and the r^{th} event comes at t

Linear Scaling

Definition

Let $Y = aX + b$, where X, Y are RV and a, b are constants

- $f_Y(u) = f_X\left(\frac{u-b}{a}\right) \times \frac{1}{a}$
- Example - $X \sim Uniform(0,1)$

- $F_Y(u) = P\{aX + b < u\}$
- $f_Y(u) = F'_Y(u) =$
- $E[Y] =$

- $Var(Y) =$ $\sigma_Y =$

Example

Consider the temperature in Champaign

- X denotes the temperature in C (Celcius)
- Y denotes it in F (Fahrenheit)
- $Y = (1.8)X + 32$

- Express f_Y in terms of f_X
- Find f_Y if $X \sim Uniform(15, 20)$

Example

Let $X \sim Uniform(a, b)$. Find the pdf of standardized RV $\frac{X-\mu_X}{\sigma_X}$

- Recall the variance of $Uniform(0,1)$ is 12

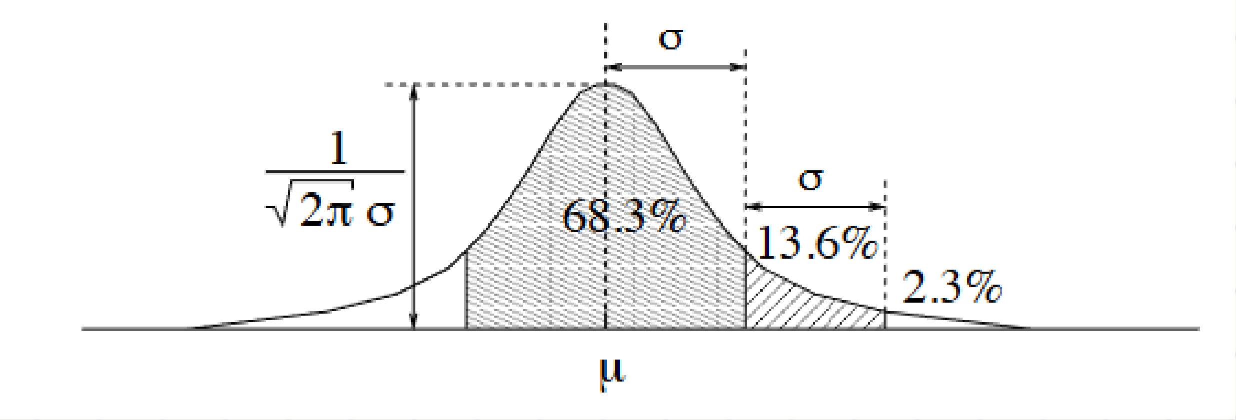
Gaussian (Normal) Distribution

Definition

A normal distribution is defined by μ_X and σ_X^2, Let $X \sim N(\mu, \sigma^2)$

- $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$

- Usage – model “Sum of many small independent events”
- E.g. Sum of many binomial distributions



Standard normal distribtuion

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$$

$$X \sim N(\mu = 0, \sigma^2 = 1) \sim N(0,1)$$

- $\Phi(u) \triangleq F_X(u) =$
- $Q(u) = 1 - \Phi(u)$

Scaling the Gaussian RV

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$$

$$X \sim N(\mu = 0, \sigma^2 = 1) \sim N(0,1)$$

- $Y = \sigma X + \mu$
- $f_Y(y) =$