Last lecture

Uniform Distribution (Ch 3.3)

- Definition
- Properties

Exponential Distribution (Ch 3.4)

- Definition
- Properties
- Connection with Geometric RV

Agenda

Poisson process (Ch 3.5)

- Motivation
- Bernoulli process to Poisson process
- Definition
- Properties

Erlang Distribution (Ch 3.5.3)

- Definition
- Properties

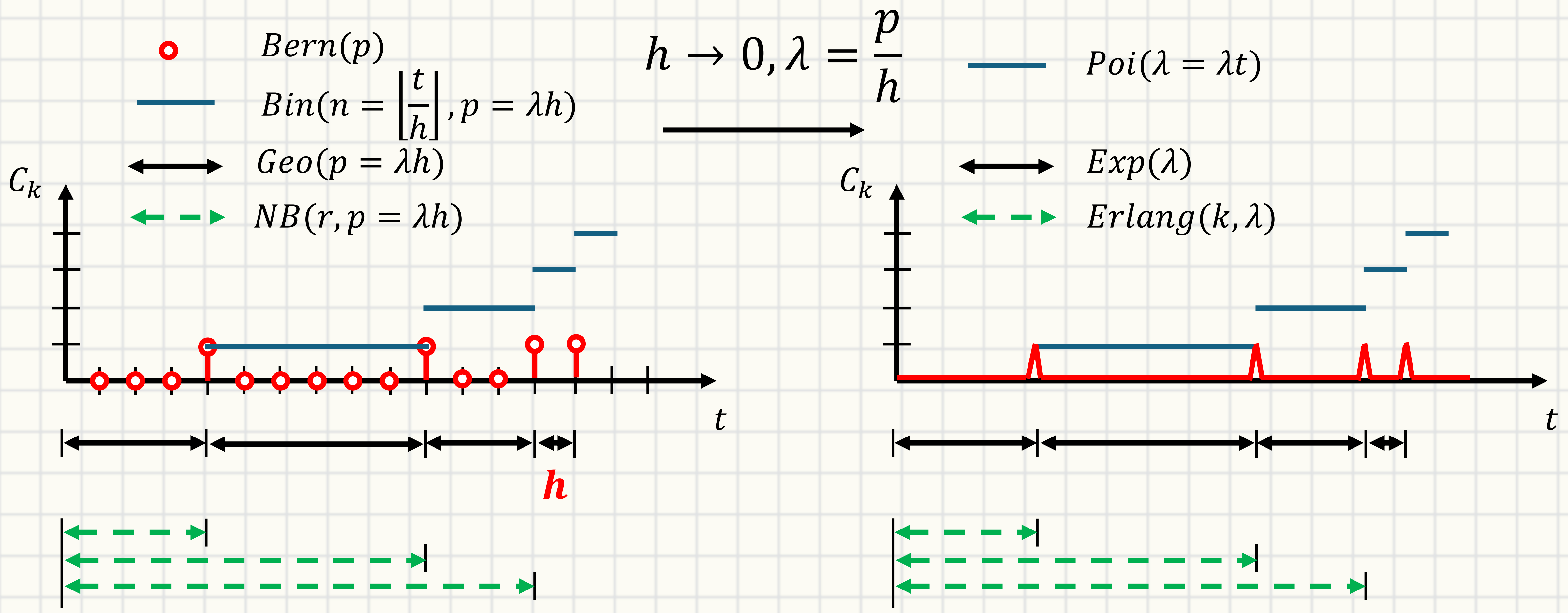
Poisson Process

Motivation

- Model the process of customer coming inside a coffee shop
- The process of incoming calls for customer service
  - If a customer (call) only comes at the   of every minutes -> Bernoulli process
  - But what if we want to model the time
  - Define a small h between Bernoulli trials

Bernoulli Process $\xrightarrow{h \to 0, \lambda = \frac{p}{h}}$ Poisson Process

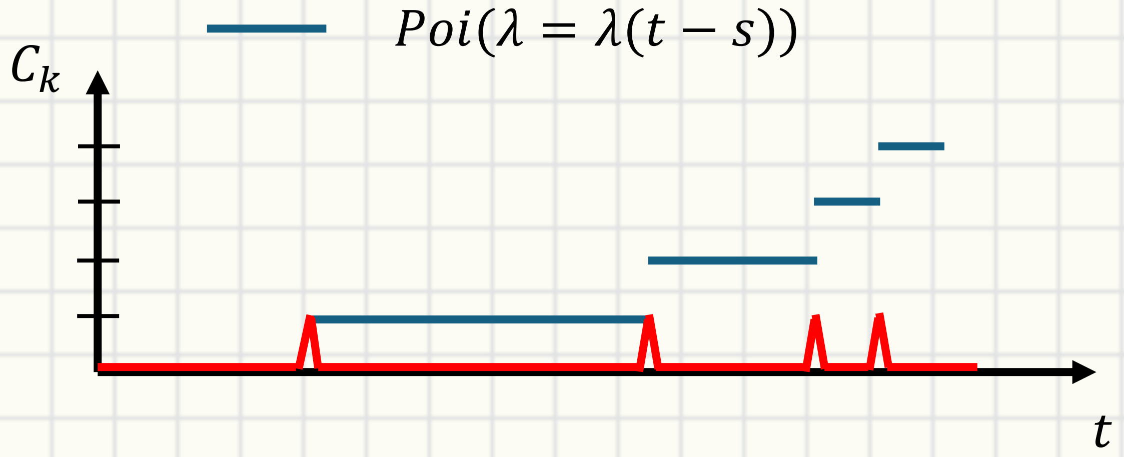
- Assume each trial takes h time to complete



Definition

A Poisson process with rate λ is a random counting process $N = (N_t : t \geq 0)$ s.t.

- $N_t - N_s$ follows Poisson distribution $Poi(\lambda = \lambda(t-s))$
- For $0 \leq t_1 \leq t_2 \ldots \leq t_k$, $N_{t_k} - N_{t_{k-1}}$ the increments are independent with each other



Example

Consider a Poisson process with rate $\lambda > 0$ in time interval $[0, T]$

- X is the total number of count during $[0, T]$
- X_1 is the count during $[0, \tau]$, $0 < \tau < T$
- X_2 is the count during $[\tau, T]$

- Solve $P\{X = n\}$, $P\{X_1 = i\}$ and $P\{X_2 = j\}$

Example

Consider a Poisson process with rate $\lambda > 0$ in time interval $[0, T]$

- X is the total number of count during $[0, T]$
- X_1 is the count during $[0, \tau]$, $0 < \tau < T$
- X_2 is the count during $[\tau, T]$

- Let $n = i + j$
- Solve $P(X = n | X_1 = i)$

Example

Consider a Poisson process with rate $\lambda > 0$ in time interval $[0, T]$

- X is the total number of count during $[0, T]$
- X_1 is the count during $[0, \tau]$, $0 < \tau < T$
- X_2 is the count during $[\tau, T]$

- Let $n = i + j$
- Solve $P(X_1 = i | X = n)$

Example

Calls arrive to a support center at rate $\lambda = 2$ calls per minute.
Let N_t denotes the number of calls until time t (mins)

- $N_t \sim$
- $P_{N_t}(k) =$
- $P\{$ No calls arrive in the first 3.5 minutes $\}$
- $P\{$ The third call arrives after time t = 3.$\}$

Erlang Distribution

Definition

Let T_r denotes the time of r^{th} count of a Poisson process

- $T_r = \sum_{i=1}^{r} U_i \, , U_i \sim Exp(\lambda)$
- $F_{T_r}^{c}(\mathrm{t}) = \mathrm{P}\{\mathrm{T_r} > \mathrm{t}\}$: "At most $r-1$ count by time t"
- $F_{T_r}^{c}(\mathrm{t}) =$
- $f_{T_r}(t) = -\frac{dF_{T_r}(t)}{dt} =$