Last lecture

Cumulative Distribution Function (Ch 3.1)

- Examples
- CDF to PMF and probabilistic density function (PDF)

Continuous RV & Probability Density Function (Ch 3.2)

- Definition
- Facts

Agenda

Uniform Distribution (Ch 3.3)

- Definition
- Properties

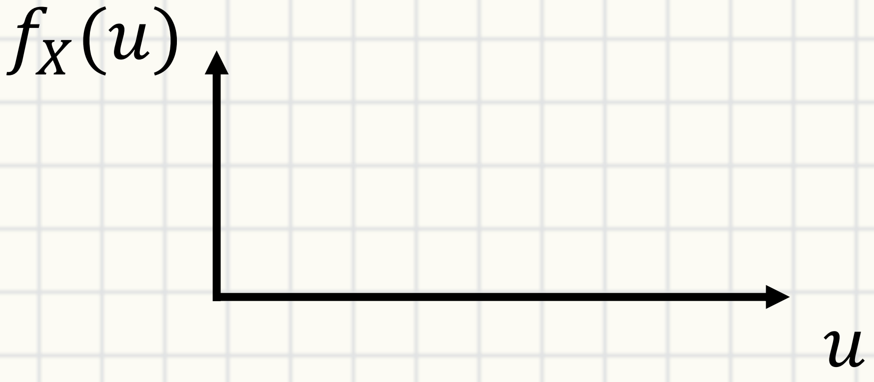
Exponential Distribution (Ch 3.4)

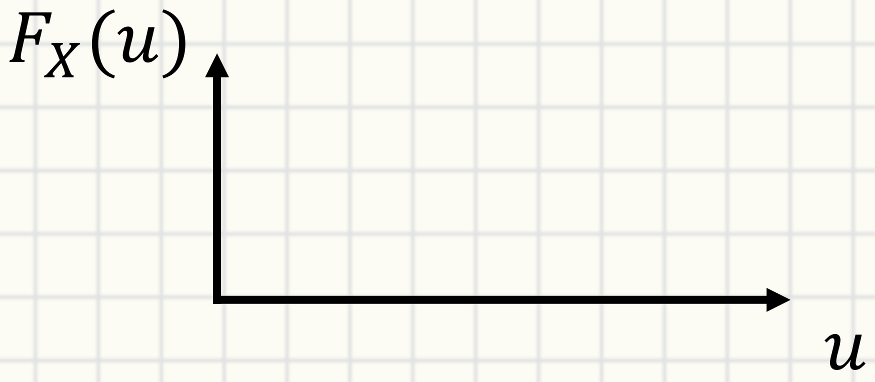
- Definition
- Properties
- Connection with Geometric RV

Uniform Distribution

Uniform Distribution

$$f_X(u) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq u \leq b \\ 0 & \text{else} \end{cases}$$





Properties

$$f_X(u) = \begin{cases} \frac{1}{b-a} & \text{if } a \le u \le b \\ 0 & \text{else} \end{cases}$$

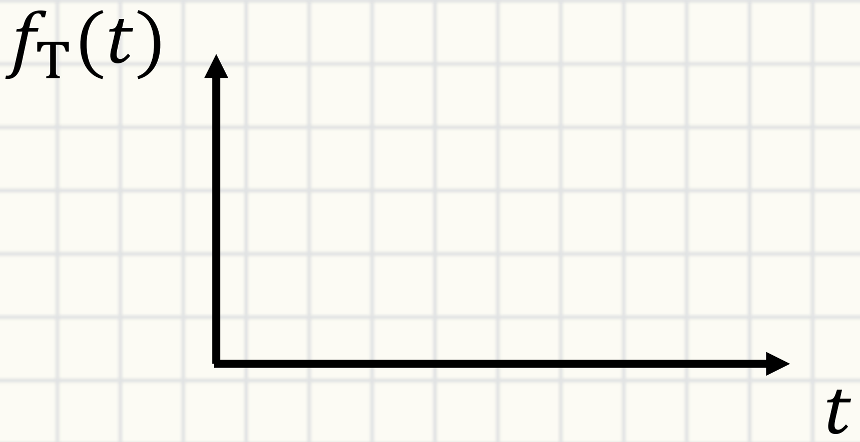
- $E[X] = \int_{-\infty}^{\infty} u f_X(u)\, du =$
- $E[X^2] = \int_{-\infty}^{\infty} u^2 f_X(u)\, du =$
- $Var(X) =$
- Special case, when $(a, b) = (0,1)$
    - k^{th} moment $E[X^k] =$
    - $Var(X) =$

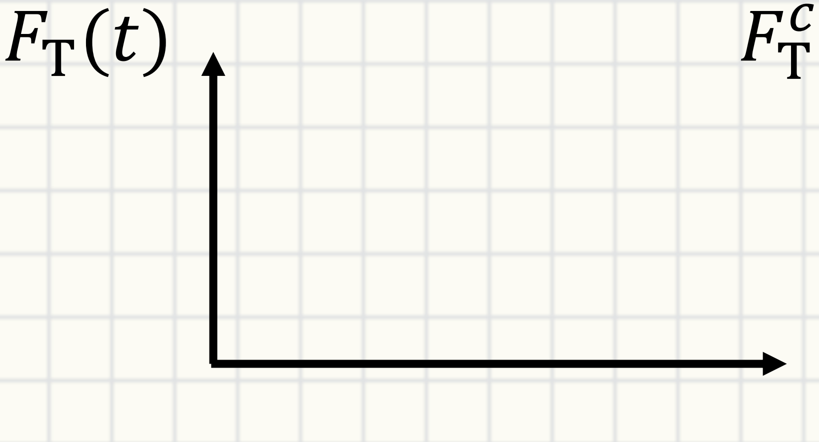
Exponential Distribution

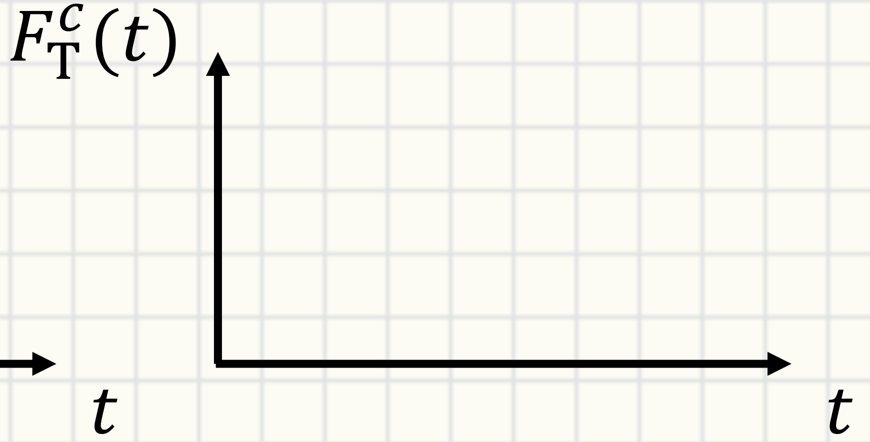
Exponential Distribution

Motivation – System life for failure rate λ

$$f_T(t) = \begin{cases} \lambda e^{-\lambda t} & \text{if } t \geq 0 \\ 0 & \text{else} \end{cases} \qquad F_T(t) = \begin{cases} 1 - e^{-\lambda t} & \text{if } t \geq 0 \\ 0 & \text{else} \end{cases}$$







Properties

$$f_T(t) = \begin{cases} \lambda e^{-\lambda t} & \text{if } t \geq 0 \\ 0 & \text{else} \end{cases}$$

- $E[T^n] = \int_0^\infty t^n \lambda e^{-\lambda t} dt$
  $= -t^n e^{-\lambda t} \Big|_0^\infty + \int_0^\infty n t^{n-1} e^{-\lambda t} dt$
  $= 0 + \frac{n}{\lambda} \int_0^\infty t^{n-1} \lambda e^{-\lambda t} dt = \frac{n}{\lambda} E[T^{n-1}]$

- $E[T] = \frac{1}{\lambda}$ $\qquad E[T^2] = \frac{2}{\lambda^2}$ $\qquad E[T^n] = \frac{n!}{\lambda^n}$

- $Var(T) =$

Examples

$$F_T(t) = \begin{cases} 1 - e^{-\lambda t} & \text{if } t \geq 0 \\ 0 & \text{else} \end{cases}$$

Let $T \sim Exp(\lambda = ln2)$, find $P\{T \geq t\}$ and $P(T \leq 1 | T \leq 2)$

Memoryless Property

$$F_T(t) = \begin{cases} 1 - e^{-\lambda t} & \text{if } t \geq 0 \\ 0 & \text{else} \end{cases}$$

$$P\{T \geq t\} = e^{-\lambda t}$$

- $P\{T \geq s + t | T \geq s\} =$
- If T is the system lifetime

Connecting Exp with Geo

$$F_T(t) = \begin{cases} 1 - e^{-\lambda t} & \text{if } t \geq 0 \\ 0 & \text{else} \end{cases}$$

Summary - $F_L(\lfloor \frac{c}{h} \rfloor) \to F_T(c)$ when $h \to 0$

- $L \sim Geo(p = \lambda h)$
- $T \sim Exp(\lambda = \lambda)$

A lightbulb of average lifetime 1000hrs

- Failed hour = $L \sim Geo(p = \frac{1}{1000})$
- Let's assume it will only fail at start of each ticks h hours (e.g., sec, $h = 1/3600$)
- Failed ticks = $L_h \sim Geo(p_h = \frac{1}{1000} \times h)$

Connecting Exp with Geo

$$F_T(t) = \begin{cases} 1 - e^{-\lambda t} & \text{if } t \geq 0 \\ 0 & \text{else} \end{cases}$$

A lightbulb of average lifetime 1000hrs

- Failed hour = $L \sim Geo(p = \frac{1}{1000})$
- Let's assume it will only fail at start of each ticks h hours (e.g., sec, $h = 1/3600$)
- Failed ticks = $L_h \sim Geo(p = \frac{1}{1000} \times h)$

- $P\{L_h h > c\} = P\left\{L_h > \frac{c}{h}\right\} = F^c_{L_h}\left(\left\lfloor\frac{c}{h}\right\rfloor\right) = (1-p)^{\left\lfloor\frac{c}{h}\right\rfloor}$

$$= (1-\lambda h)^{\left\lfloor\frac{c}{h}\right\rfloor} = \lim_{h\to 0} -e^{-\lambda c} = F^c_T(c)$$