Last lecture

Reliability & Union Bound(Ch 2.12)

- Examples – network outage
- Examples – Array code

Continuous RV (Ch 3)

- Motivation
- Cumulative Distribution Function (Ch 3.1)
- Examples

Agenda

Cumulative Distribution Function (Ch 3.1)

- Examples
- CDF to PMF and probabilistic density function (PDF)

Continuous RV & Probability Density Function (Ch 3.2)

- Definition
- Facts

Examples

- Find all u where $P\{X = u\} > 0$
- Find $P\{X \leq 0\}$
- Find $P\{X < 0\}$

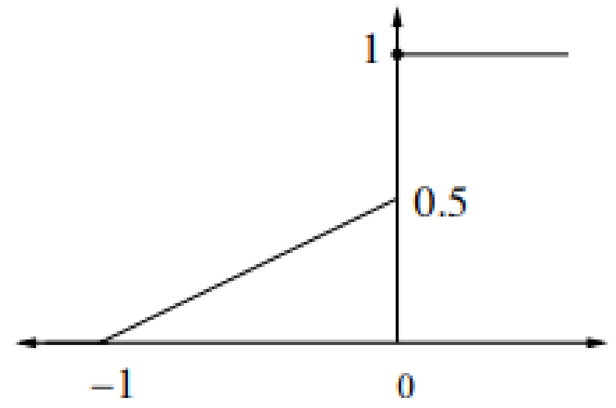
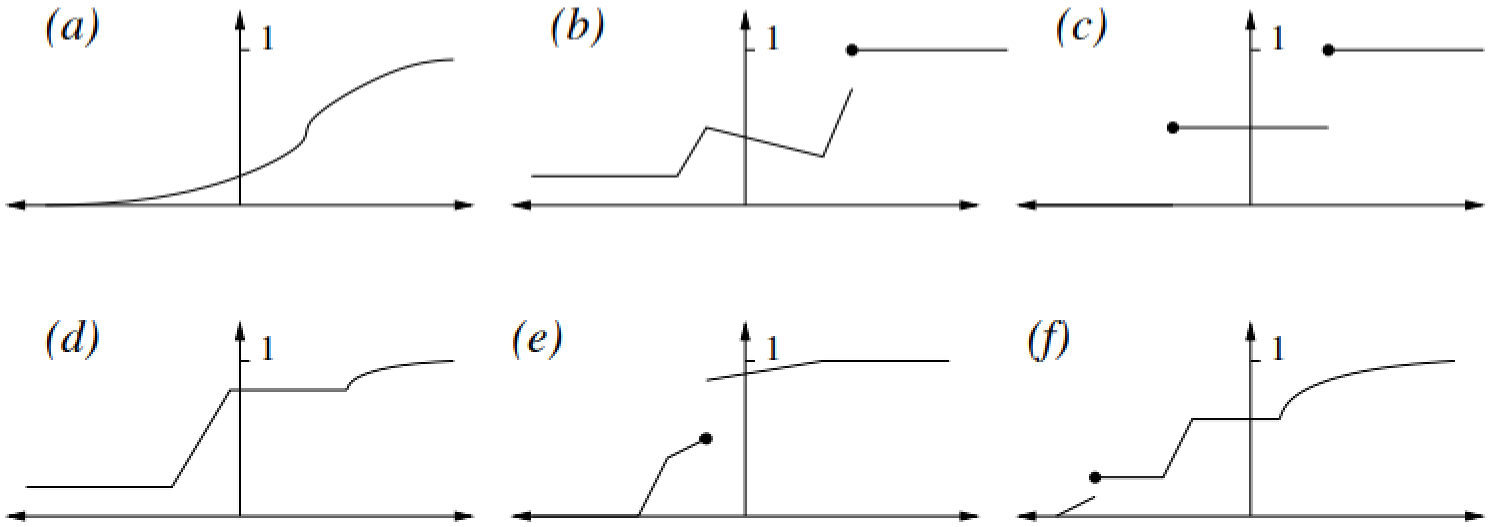


Figure 3.2: An example of a CDF.

CDF Properties

A function F is a CDF of some RV iif

- F is none-decreasing
- $\lim_{c \to \infty} F(c) = 1$ and $\lim_{c \to -\infty} F(c) = 0$
- F is right-continuous



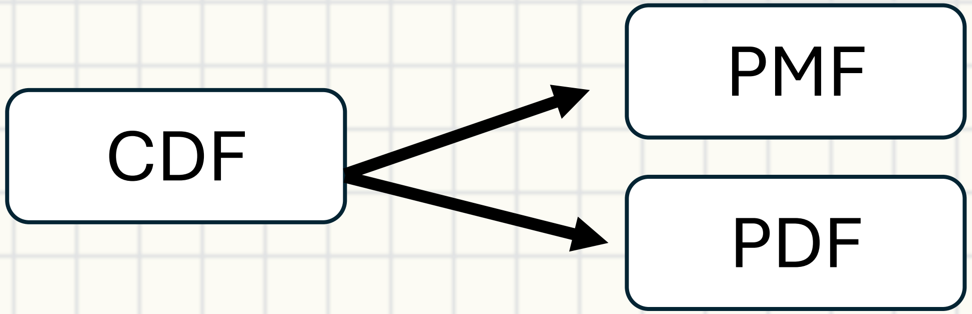
CDF to PMF and PDF

For discrete RV

- $F_X(c) =$
- $p_X(u) =$

For continuous RV

- $F_X(c) =$
- $f_X(u) =$
- $f_X(u)$ is called " " (PDF)



Continuous RV (Ch 3.2)

Continuous RV and PDF

X is a continuous RV if its pdf f_X follows

- $F_X(c) = \int_{-\infty}^{c} f_X(u)du$ for all $c \in \mathbb{R}$
- Support – $\{u: f_X(u) > 0\}$
- If $F_X(c)$ is continuous and differentiable, $f_X = F_X{}'$
  - Since there is no jump in $F_X(c)$, $P\{X = c\} =$
- $P\{a < X \leq b\} = F_X(b) - F_X(a) =$
- $\int_{-\infty}^{\infty} p_X(u)du$

Why P "density" F

By definition, $f_X = F_X'$

$$f_X(u_0) = \lim_{h \to 0} \frac{F(u_0 + h) - F(u_0)}{h}$$

$$f_X(u_0) = \lim_{h \to 0} \frac{F(u_0) - F(u_0 - h)}{h}$$

$$f_X(u_0) = \lim_{h \to 0} \frac{F(u_0 + h) - F(u_0 - h)}{2h}$$

- Let $\epsilon = 2h > 0$
- $f_X(u_0) = \lim_{h \to 0} \frac{F\left(u_0 + \frac{\epsilon}{2}\right) - F\left(u_0 - \frac{\epsilon}{2}\right)}{\epsilon}$
- $P\left\{u_0 - \frac{\epsilon}{2} < X < u_0 + \frac{\epsilon}{2}\right\} = \epsilon f_X(u_0) + O(\epsilon)$
- "Density of the probability"

Expectation and Variance

- $\mu_X = E[X] = \int_{-\infty}^{\infty} u f_X(u) du$
- LOTUS still applies, $E[g(x)] = \int_{-\infty}^{\infty} g(u) f_X(u) du$
- E.g. $E[aX^2 + bX + c] = aE[X^2] + bE[X] + c$

- $\sigma_X^2 = Var(X) = E[(X - \mu_X)^2] = E[X^2] - \mu_X^2$

Example

- $f_X(u) = \begin{cases} A(1-u^2) & if \quad -1 \le u \le 1 \\ 0 & else \end{cases}$

- Find A, $P\{-0.5 < X < 1.5\}$, F_X, μ_X, σ_X^2