Last lecture

Binary Hypothesis Testing (Ch 2.11)

- Maximum A Posteriori (MAP) decision rule examples

Reliability & Union Bound(Ch 2.12.1)

- Definition
- Examples – network outage

Agenda

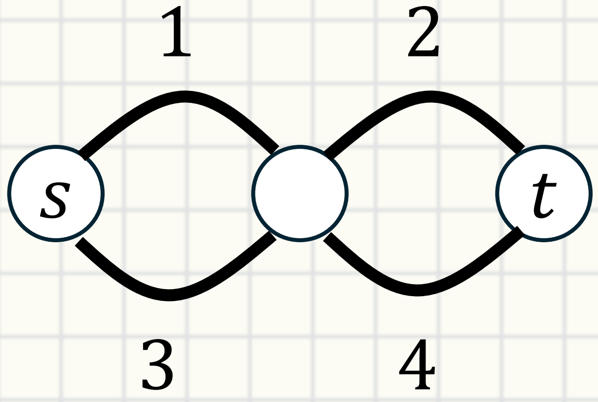
Reliability & Union Bound(Ch 2.12)

- Examples – network outage
- Examples – Array code

Continuous RV (Ch 3)

- Motivation
- Cumulative Distribution Function (Ch 3.1)
- Examples

Example – Network outage



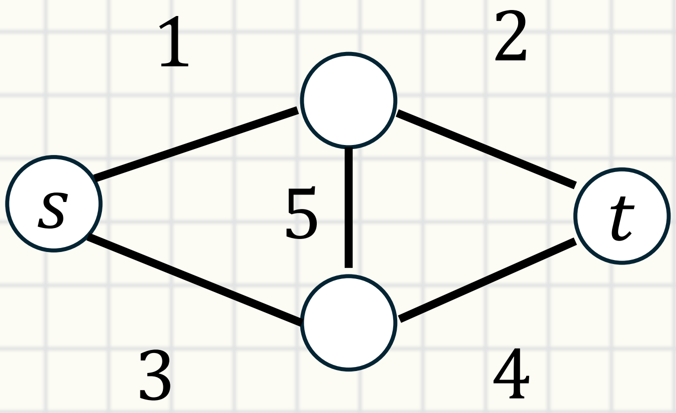
Compute $P(F)$

- $P(F) = P(F_L \cup F_R)$

Exact probability $p_k = 0.001$

Union bound $p_k = 0.001$

Slido



$P(F) =$

4639950

$(a)\ p_1p_3 + p_2p_4 - p_1p_2p_3p_4$

$(b)\ p_5(p_1p_3 + p_2p_4 - p_1p_2p_3p_4)$

$(c)\ p_5(p_1p_3 + p_2p_4 - p_1p_2p_3p_4) + q_5(p_1 + p_2 - p_1p_2)(p_3 + p_4 - p_3p_4)$

$(d)\ p_1p_3 + p_2p_4 - p_1p_2p_3p_4 + p_1q_2q_3p_4p_5 + q_1p_2p_3q_4p_5$

Array code

2D parity code

- Error detection and correction
- Last row/ column are bits
- Fill in the bits such that rows and columns are parity
- Carry $(n-1)^2$ info bits using n^2 bits
- Detect up to bits

1	0	1	0	0	1	0	
1	0	0	1	0	0	1	
0	1	0	1	1	1	0	
1	0	0	0	1	0	1	
0	0	1	1	0	1	0	
1	1	0	1	0	1	1	
0	1	0	0	1	0	1	
0	1	0	0	1	0	0	

Array code

Assume $BER = p = 10^{-3}$

Let Y denotes the number of bit errors

- $P_Y(k) =$

- Bound for undetected errors U

- $P(U) \leq P\{Y \geq 4\} \leq$

- Tighter bound?

1	0	1	0	0	1	0	1
1	0	0	1	0	0	1	1
0	1	0	1	1	1	0	0
1	0	0	0	1	0	1	1
0	0	1	1	0	1	0	1
1	1	0	1	0	1	1	1
0	1	0	0	1	0	1	1
0	1	0	0	1	0	0	0

Continuous RV

Motivation

Tired of coin toss/ win-lose?

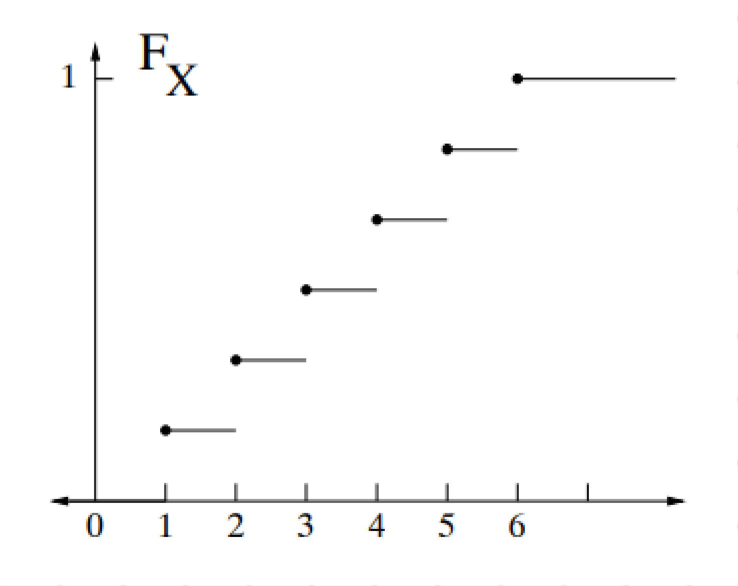
- Real-world is continuous
- Time, space, height, weight, colors, etc.
- But how do we define a continuous RV?
- Recall for prob. space $(\Omega, \mathcal{F}, P)$
  - X maps $\omega \in \Omega$ to $\mathbb{R}$ (coated die)
  - What if ω is continuous?
  - Discrete $\{\omega: X(\omega) = c\}$ -> Continuous $\{\omega: X(\omega) \leq c\}$
  - $F_X(c) = P\{\omega: X(\omega) \leq c\} = P\{X \leq c\}$

Cumulative Distribution Function (CDF

Recall for prob. space $(\Omega, \mathcal{F}, P)$

- X maps $\omega \in \Omega$ to $\mathbb{R}$ (coated die)
- PMF $\{\omega : X(\omega) = c\}$ -> CDF $\{\omega : X(\omega) \leq c\}$
- $F_X(c) = P\{\omega : X(\omega) \leq c\} = P\{X \leq c\}$

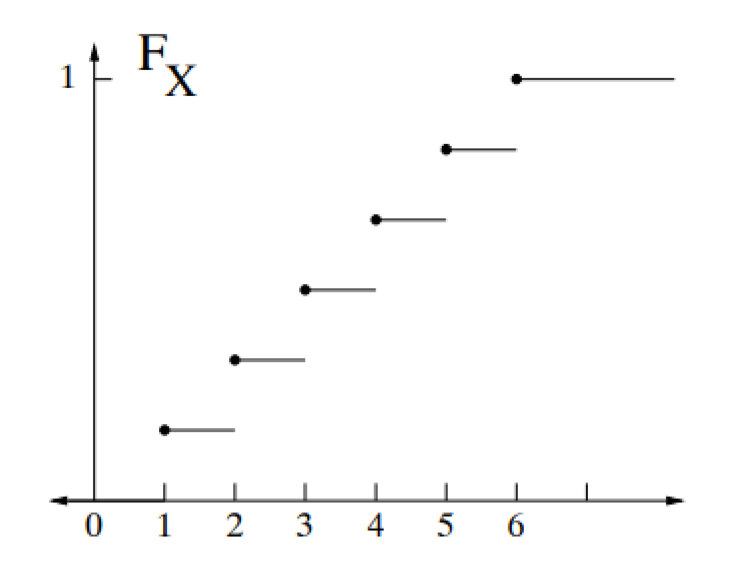
CDF of a fair die roll



Recall – Left limit and Right limit

- $F_X(x-) = \lim_{\substack{y \to x \\ y<x}} F_X(y)$ $\qquad F_X(x+) = \lim_{\substack{y \to x \\ y>x}} F_X(y)$
- $F_X(x) \triangleq F_X(x+)$
- $F_X(2+) =$
- $\Delta F_X(x) = F_X(x) - F_X(x-)$

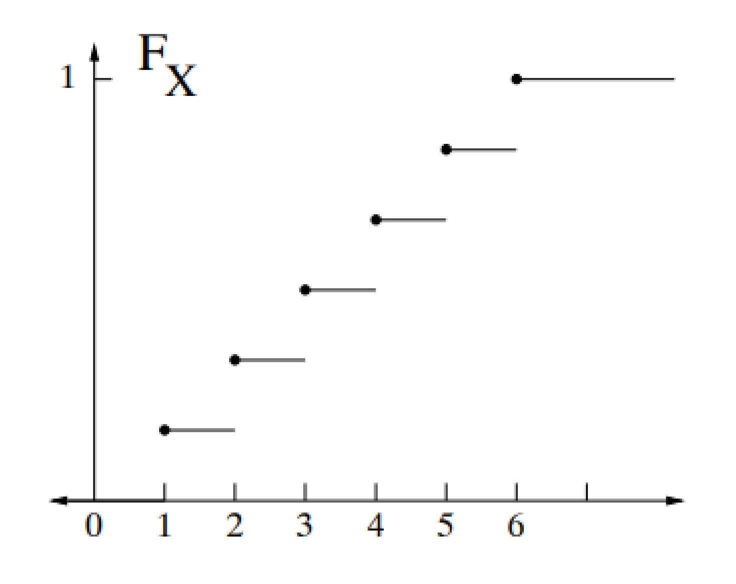
- $P\{X \in (a,b]\} =$



Recall – Left limit and Right limit

- $F_X(x-) = \lim_{\substack{y \to x \\ y < x}} F_X(y)$ $\qquad$ $F_X(x+) = \lim_{\substack{y \to x \\ y > x}} F_X(y)$
- $F_X(x) \triangleq F_X(x+)$
- $F_X(2+) =$
- $\Delta F_X(x) = F_X(x) - F_X(x-)$

- $P\{X \in (a, b]\} =$



Examples

- Find all u where $P\{X = u\} > 0$
- Find $P\{X \leq 0\}$
- Find $P\{X < 0\}$

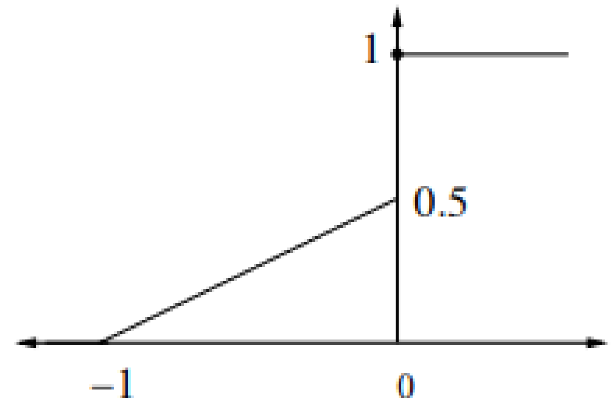


Figure 3.2: An example of a CDF.

CDF Properties

A function F is a CDF of some RV iif

- F is none-decreasing
- $\lim_{c \to \infty} F(c) = 1$ and $\lim_{c \to -\infty} F(c) = 0$
- F is right-continuous