# Last lecture

Binary Hypothesis Testing (Ch 2.11)

- Maximum A Posteriori (MAP) decision rule examples

Reliability & Union Bound(Ch 2.12.1)

- Definition
- Examples – network outage

# Agenda

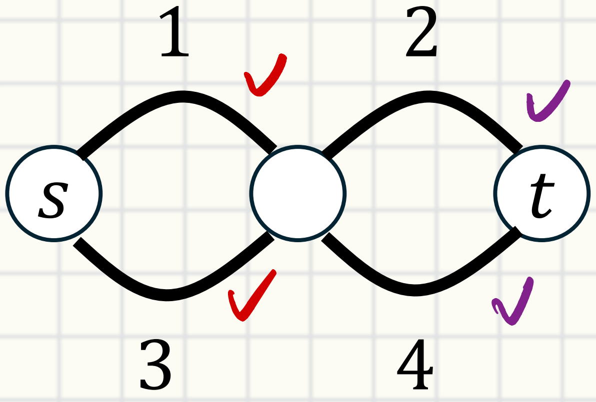
Reliability & Union Bound(Ch 2.12)

- Examples – network outage
- Examples – Array code

Continuous RV (Ch 3)

- Motivation
- Cumulative Distribution Function (Ch 3.1)
- Examples

# Example – Network outage




$p_i \triangleq P(\text{link } i \text{ fails})$

Compute $P(F)$

- $P(F) = P(\underbrace{F_L}_{p_1 p_3} \cup \underbrace{F_R}_{p_2 p_4})$

Exact probability $p_k = 0.001$

$P(F) = P(F_L) + P(F_R) - P(F_L \cap F_R)$

$= 10^{-6} + 10^{-6} - 10^{-12}$

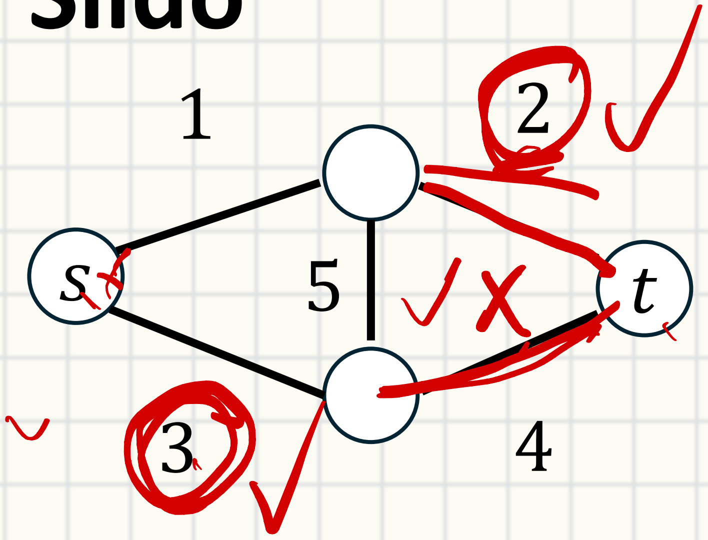
$\approx 2 \times 10^{-6}$

Union bound $p_k = 0.001$

$P(F_L \cup F_R) \leq P(F_L) + P(F_R)$

$= 2 \times 10^{-6}$

# Slido




$q_i = 1 - p_i$

$$P(F) =$$

# 4639950

(a) $p_1p_3 + p_2p_4 - p_1p_2p_3p_4$ ⇒ link 5 snaps bridge node

(b) $\overset{q_5}{\cancel{p_5}}(p_1p_3 + p_2p_4 - p_1p_2p_3p_4)$ ⇒ only when 5 is up

(c) $\underset{q_5}{\cancel{p_5}}(p_1p_3 + p_2p_4 - p_1p_2p_3p_4) + \underset{p_5}{\cancel{q_5}}(p_1 + p_2 - p_1p_2)(p_3 + p_4 - p_3p_4)$ ✓

(d) $p_1p_3 + p_2p_4 - p_1p_2p_3p_4 + p_1q_2q_3p_4p_5 + q_1p_2p_3q_4p_5$ ✓

# Array code

2D parity code

- Error detection and correction
- Last row/ column are parity bits — depends on info bits
- Fill in the bits such that rows and columns are even parity — even # of 1s
- Carry $(n-1)^2$ info bits using $n^2$ bits
- Detect up to 3 bits

x bit flip

4 bits counter example

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 0 | ~~1~~ 0 | 0 | 0 | ~~1~~ 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | ~~0~~ 1 | 0 | ~~0~~ 1 | 1 | 0 | 1 | 1 |
| 0 | 0 | ~~1~~ 0 | 1 | 0 | ~~1~~ 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | $\boxed{0}$ |

X

# Array code

Assume $BER = p = 10^{-3}$

Let $Y$ denotes the number of bit errors

- $P_Y(k) = \binom{64}{k} p^k (1-p)^{64-k}$
- Bound for undetected errors $U$
- $P(U) \le P\{Y \ge 4\} \le 1 - \sum_{k=0}^{3} P_Y(k)$
- Tighter bound?

$\binom{8}{2}\binom{8}{2} p^4 (1-p)^{60} + P\{Y \ge 6\}$

8

8

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |

# Continuous RV

# Motivation

Tired of coin toss/ win-lose?

- Real-world is continuous
- Time, space, height, weight, colors, etc.
- But how do we define a continuous RV?
- Recall for prob. space $(\Omega, \mathcal{F}, P)$
  - $X$ maps $\omega \in \Omega$ to $\mathbb{R}$ (coated die)
  - What if $\omega$ is continuous? $P_X(c)$ $F_X(c)$
  - Discrete $\{\omega: X(\omega) = c\}$ -> Continuous $\{\omega: X(\omega) \leq c\}$
  - $F_X(c) = P\{\omega: X(\omega) \leq c\} = P\{X \leq c\}$

CDF.

# Cumulative Distribution Function (CDF)

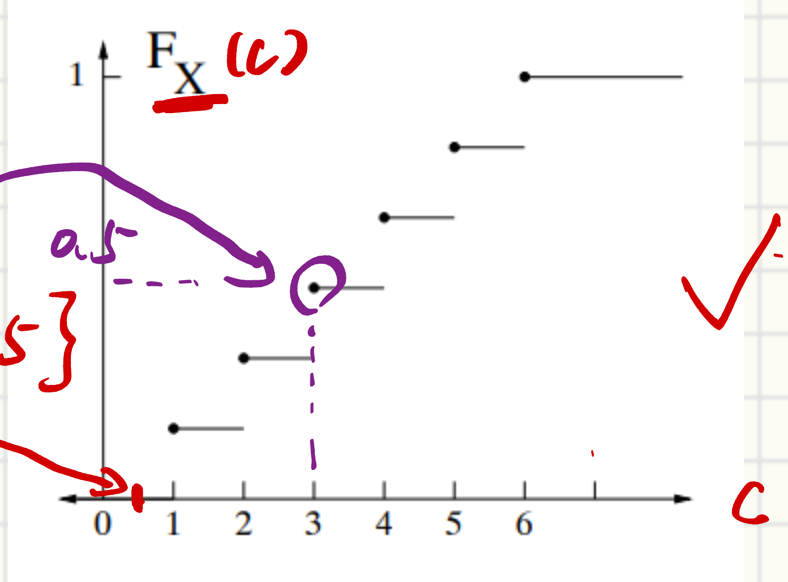
Recall for prob. space $(\Omega, \mathcal{F}, P)$

- $X$ maps $\omega \in \Omega$ to $\mathbb{R}$ (coated die)
- PMF $\{\omega: X(\omega) = c\}$ -> CDF $\{\omega: X(\omega) \leq c\}$
- $F_X(c) = P\{\omega: X(\omega) \leq c\} = P\{X \leq c\}$

CDF of a fair die roll

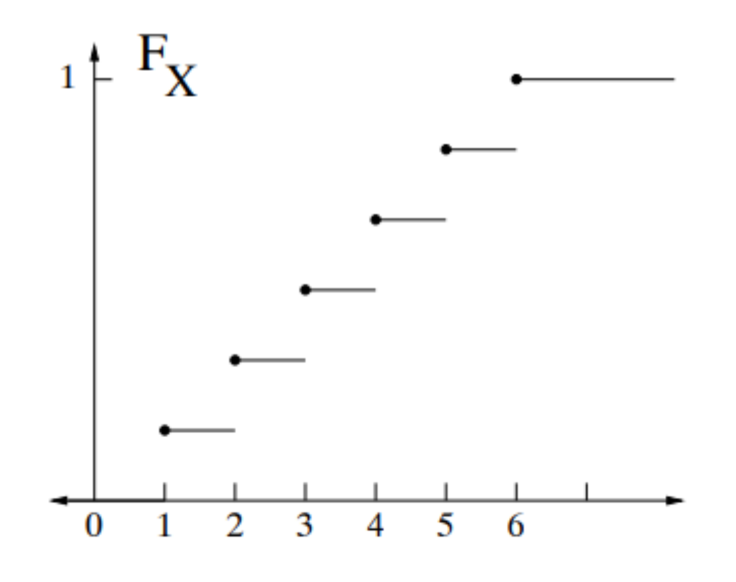
$F_X(3) = P\{X \leq 3\}$

$F_X(0.5) = P\{X \leq 0.5\}$



# Recall – Left limit and Right limit

- $F_X(x-) = \lim_{\substack{y \to x \\ y<x}} F_X(y)$ $\qquad$ $F_X(x+) = \lim_{\substack{y \to x \\ y>x}} F_X(y)$
- $F_X(x) \triangleq F_X(x+)$
- $F_X(2+) =$ $F_X(2) = \frac{2}{6} = \frac{1}{3}$
- $\Delta F_X(x) = F_X(x) - F_X(x-)$ $= P_X(x)$
- $P\{X \in (a,b]\} =$ $F_X(b) - F_X(a)$



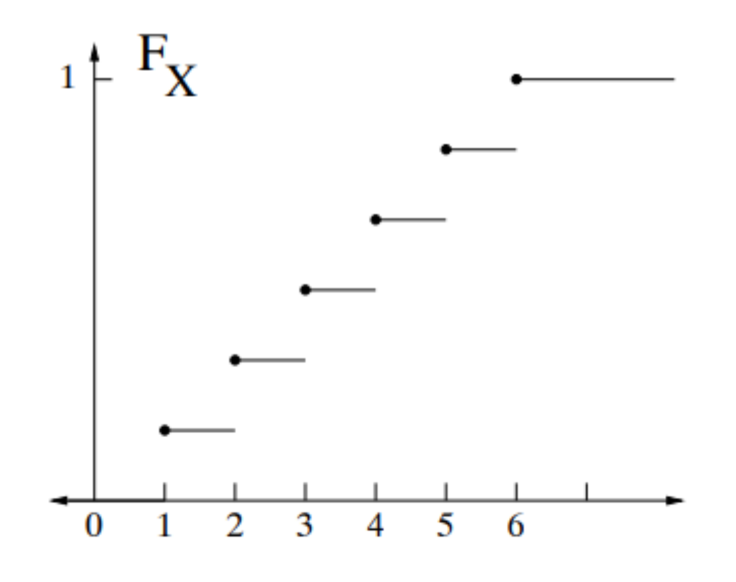
# Recall – Left limit and Right limit

- $F_X(x-) = \lim_{\substack{y \to x \\ y < x}} F_X(y)$

$F_X(x+) = \lim_{\substack{y \to x \\ y > x}} F_X(y)$

- $F_X(x) \triangleq F_X(x+)$
- $F_X(2+) =$
- $\Delta F_X(x) = F_X(x) - F_X(x-)$

- $P\{X \in (a, b]\} =$



# Examples

- Find all $u$ where $P\{X = u\} > 0$
- Find $P\{X \leq 0\}$ $= F(0) = 1$
- Find $P\{X < 0\}$

$= 1 - 0.5 = 0.5$

$u = 0$ $P(u) =$

$F(0) - F(0-)$

$= 0.5$

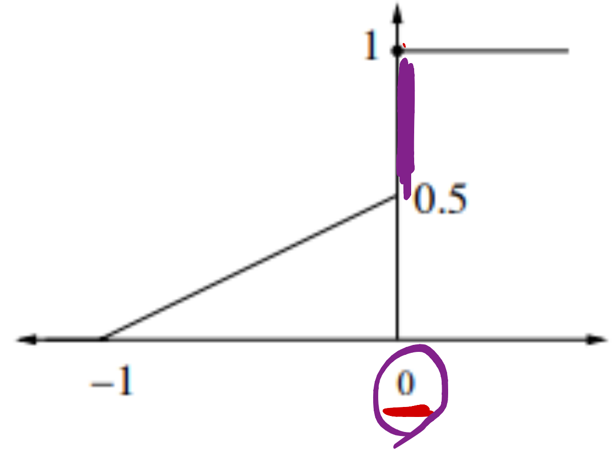



Figure 3.2: An example of a CDF.