Last lecture

Binary Hypothesis Testing (Ch 2.11)

- Likelihood table
- Maximum likelihood (ML) decision rule
- Maximum A Posteriori (MAP) decision rule

Agenda

Binary Hypothesis Testing (Ch 2.11)

- Maximum A Posteriori (MAP) decision rule examples

Reliability & Union Bound(Ch 2.12.1)

- Definition
- Examples – network outage

Likelihood table to joint probability

Assume $\pi_1 = P(H_1) = 0.2, \pi_0 =$

- Decide on joint probability is same as posterior probability

- MAP rule = LRT rule with $\tau =$

$P(X\|H)$	$X=0$	$X=1$	$X=2$	$X=3$
H_1	0	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

$P(H,X)$	$X=0$	$X=1$	$X=2$	$X=3$
H_1				
H_0				

Example

X: Draw a coin from the bag and toss it 5 times

- Likelihood table
- Joint probability table
- Describe ML and MAP rule, compute
  - p_{false_alarm}
  - p_{miss}
  - p_e

$H_1: p = \frac{2}{3}$ coin

$H_0: p = \frac{1}{2}$ coin

Example

$H_1: p = \frac{2}{3}$ coin

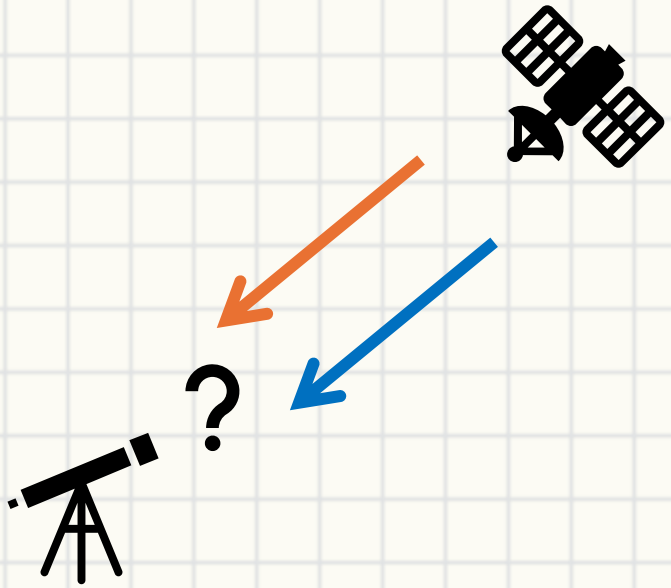
$H_0: p = \frac{1}{2}$ coin

Example

Receive on-off keying (OOK) signal from a deep space Tx.

X: # of photons observe from a telescope

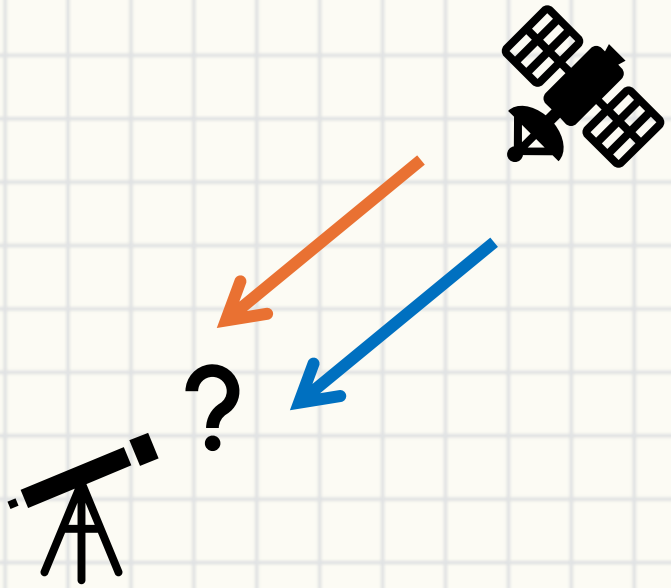
- $\lambda = 6$ If it's ON
- $\lambda = 2$ If it's OFF
- $\frac{\pi_0}{\pi_1} = 5$
- Describe ML and MAP rule, compute
  - p_{false_alarm}
  - p_{miss}
  - p_e



$H_1: Pois(\lambda = 6)$

$H_0: Pois(\lambda = 2)$

Example



$H_1: Pois(\lambda = 6)$

$H_0: Pois(\lambda = 2)$

Reliability / Union Bound

Motivation and Definition

Reliability

- How likely a system will fail?
  - Each subsystem fail with probability
  - If sub-systems fail in some pattern, the system fails

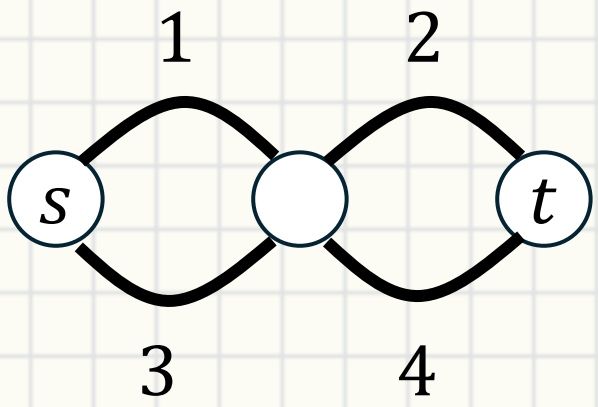
Union Bound

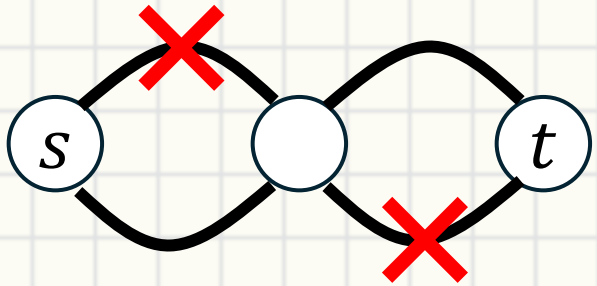
- Bounds for of small probability events
- $P(A \cup B) \leq$
- $P(A_1 \cup A_2 \cup \cdots \cup A_m) \leq$
- Bound is at most far from the actual value

Example – Network outage

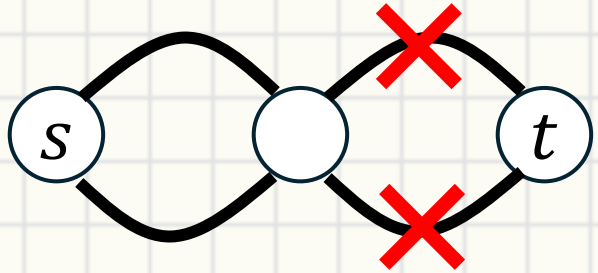
A $s-t$ network consists of nodes source (s), terminal (t), other nodes, and links.

- Each link k fails independently with small probability p_k
- Network outage event F occurs if
  - For any path from s to t, there is at least one link in the path fails



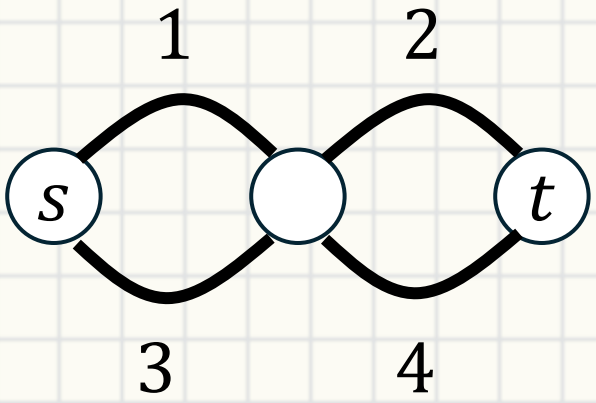


$x_1 \notin F$



$x_2 \notin F$

Example – Network outage



Compute $P(F)$

- $P(F) = P(F_L \cup F_R)$

Exact probability $p_k = 0.001$

Union bound $p_k = 0.001$