Last lecture

Markov and Chebychev inequalities (Ch 2.9)

- Chebychev inequality
- Confidence interval

Binary Hypothesis Testing (Ch 2.11)

- Definition
- Likelihood table

Agenda

Binary Hypothesis Testing (Ch 2.11)

- Likelihood table
- Maximum likelihood decision rule

Maximum Likelihood Table

- Table showing of two hypotheses

	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	0	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

- Decision rule can be shown on the table

False alarm and missing

	$X=0$	$X=1$	$X=2$	$X=3$
H_1	0	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

- $P_{false\ alarm} =$
- $P_{miss} =$

	$X=0$	$X=1$	$X=2$	$X=3$
H_1	0	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

Maximum Likelihood (ML) decision rule

Pick whichever is higher per column!

	$X=0$	$X=1$	$X=2$	$X=3$
H_1	$p_1(0)=0$	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

Likelihood Ratio Test (LRT) $\Lambda(k)=\frac{p_1(k)}{p_0(k)}$

A LRT with threshold τ:

Maximum Likelihood (ML) decision rule

Pick whichever is higher per column!

	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	0	0.1	<u>0.3</u>	<u>0.6</u>
H_0	<u>0.4</u>	<u>0.3</u>	0.2	0.1

What's the problem?

Maximum A Posteriori (MAP) decision rule

Let's pick based on $P(H|X)$ or joint probability $P(H,X)$

- Pick the higher $P(H,X)$ per column
- Lowest total error rate p_e

- But how do we get the joint probability?
- Recall conditional probability

$$P(A,B) =$$

$$P(H_1, X = k) =$$

$P(H,X)$	$X=0$	$X=1$	$X=2$	$X=3$
H_1	0	0.02	0.06	0.12
H_0	0.32	0.24	0.16	0.08

Prior and Posterior

$$P(H_1, X = k) = P(X = k|H_1)P(H_1)$$

- $P(H_1) \triangleq \pi_1$: “ “, probability assumed before observation
- $P(H_0) \triangleq \pi_0$
- Bayes’ rule revisited (Param/Hypothesis vs. Observation)

$$P(H|X) = \frac{P(X|H)P(H)}{P(X)}$$

Terminologies

Exam Safe

$$P(\theta|X) = \frac{P(X|\theta)P(\theta)}{P(X)}$$

Likelihood	$P(X\|\theta)$	Tractable, well defined	Toss $p = 0.3$ coin
Posterior	$P(\theta\|X)$	Typical goal	$x_{1:3} = \{H, T, T\}, p = ?$
Prior	$P(\theta)$	Domain knowledge	# coins $p = 0.3$ in world
Evidence	$P(X)$	Approx. with large data	# H in world

Likelihood table to joint probability

Assume $\pi_1 = P(H_1) = 0.2, \pi_0 =$

- Decide on joint probability is same as posterior probability

- MAP rule = LRT rule with $\tau =$

$P(X\|H)$	$X=0$	$X=1$	$X=2$	$X=3$
H_1	0	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

$P(H,X)$	$X=0$	$X=1$	$X=2$	$X=3$
H_1				
H_0				

Example

X: Draw a coin from the bag and toss it 5 times

- Likelihood table
- Joint probability table
- Describe ML and MAP rule, compute
  - p_{false_alarm}
  - p_{miss}
  - p_e

$H_1: p = \frac{2}{3}$ coin

$H_0: p = \frac{1}{2}$ coin