Last lecture

Markov and Chebychev inequalities (Ch 2.9)

- Chebychev inequality
- Confidence interval

Binary Hypothesis Testing (Ch 2.11)

- Definition
- Likelihood table

Correction

Midterm #1 range until

HWK #4 $\Rightarrow$ Poisson

NO MLE / confidence interval.

Agenda

Binary Hypothesis Testing (Ch 2.11)

- Likelihood table
- Maximum likelihood decision rule
- Maximum A Posteriori

Maximum Likelihood Table

- Table showing likelihood of two hypotheses

$\hookrightarrow H_1 \triangleq$ has tumor
$\hookrightarrow H_0 \triangleq$ has ✗ tumors.

	$X = 0$	$X = 1$	$X = 2$	$X = 3$	
$P(X \mid H_1) \rightarrow$ H_1	0	<u>0.1</u>	<u>0.3</u>	<u>0.6</u>	$1 = \sum P(X \mid H_1)$
$P(X \mid H_0) \rightarrow$ H_0	<u>0.4</u>	0.3	0.2	0.1	$\Rightarrow \sum = 1.$

If Alice has tumor, P{Alice ultrasound scan has 2 circles | H_1}

- Decision rule can be shown on the table by underlining a cell per column

False alarm and missing

Metrics for rules

	$X=0$	$X=1$	$X=2$	$X=3$
H_1	0	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

- $P_{false\ alarm} =$ $P\{\text{Claim } H_1 \mid H_0\} = 0.3 + 0.2 + 0.1 = 0.6$
  (Type I error)
- $P_{miss} =$ $P\{\text{Claim } H_0 \mid H_1\} = 0$
  (Type II error

Another policy

	$X=0$	$X=1$	$X=2$	$X=3$
H_1	0	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

$P_{false\ alarm} = 0.2 + 0.1 = 0.3$

$P_{miss} = 0 + 0.1 = 0.1$

Maximum Likelihood (ML) decision rule

Pick whichever is higher per column!

$\Lambda(1) = \frac{1}{3}$

	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	$p_1(0) = 0$	0.1	<u>0.3</u>	<u>0.6</u>
H_0	<u>0.4</u>	<u>0.3</u>	0.2	0.1

$\Lambda(0) = \frac{0}{0.4} = 0$ $\Lambda(2) = \frac{3}{2}$

<u>Likelihood Ratio Test (LRT)</u> $\Lambda(k) = \frac{p_1(k)}{p_0(k)}$ $\rightarrow$ MLE =

<u>A LRT with threshold τ:</u>

$$\begin{cases} H_1 & \text{if } \Lambda(k) > \tau \\ H_0 & \text{else} \end{cases}$$

LRT w/ $\tau = 1$

Maximum Likelihood (ML) decision rule

Pick whichever is higher per column!

? $P(H_1) \Rightarrow$

	$X=0$	$X=1$	$X=2$	$X=3$
H_1	0	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

What's the problem?

$\Rightarrow$ If $P_{\text{false alarm}} \uparrow$ $\quad P(H_0) \uparrow$

$\downarrow$ $P(\text{claim } H_1 \mid H_0)$ $\Rightarrow$ large (Type I error count)

$P_{\text{miss}} \uparrow$ $\quad P(H_1) \uparrow$

Slidos – Midterm Review and Early Feedback

Midterm Review

- This Friday class time
- Vote for the contents!

#2124287

Early Feedback

- Suggest improvement
- Anonymous by default

#9021295

Maximum A Posteriori (MAP) decision rule

Let's pick based on $P(H|X)$ or joint probability $P(H,X)$

- Pick the higher $P(H,X)$ per column
- Lowest total error rate p_e $= \frac{\text{Total error}}{\text{Total experiments (patients)}} = \sum$
- But how do we get the joint probability?
- Recall conditional probability

$$P(A,B) = P(B|A)\,P(A)$$

likelihood table

Prior

$$\underbrace{P(H_1}_{A},\underbrace{X=k}_{B}) = P(X=k|H_1) \times P(H_1)$$

$\Sigma = 1$

$P(H,X)$	$X=0$	$X=1$	$X=2$	$X=3$
H_1	0	0.02	0.06	0.12
H_0	0.32	0.24	0.16	0.08

$P(H_1) = 0.2$ $P(H_0) = 0.8$

Prior and Posterior

$$P(H_1, X = k) = P(X = k|H_1)P(H_1)$$

- $P(H_1) \triangleq \underline{\pi_1}$: " prior probability", probability assumed before observation X.
- $P(H_0) \triangleq \underline{\pi_0}$
- Bayes' rule revisited (Param/Hypothesis vs. Observation)

likelihood

$$\boxed{P(H|X) = \frac{P(X|H)P(H)}{P(X)}}$$

prior

evidence

Terminologies

Exam Safe

$$P(\theta|X) = \frac{P(X|\theta)P(\theta)}{P(X)}$$

Likelihood	$P(X\|\theta)$	Tractable, well defined	Toss $p = 0.3$ coin
Posterior	$P(\theta\|X)$	Typical goal	$x_{1:3} = \{H, T, T\}, p = ?$
Prior	$P(\theta)$	Domain knowledge	# coins $p = 0.3$ in world
Evidence	$P(X)$	Approx. with large data	# H in world

Likelihood table to joint probability

Assume $\pi_1 = P(H_1) = 0.2$, $\pi_0 = P(H_0) = 1 - \pi_1 = 0.8$

- Decide on joint probability is same as posterior probability

$P(X, H)$ $\quad P(H|X)$

- MAP rule = LRT rule with $\tau = \frac{\pi_0}{\pi_1} = \frac{0.8}{0.2} = 4$

$P(X|H_1)\cancel{P(H_1)} > \tau P(X|H_0)\cancel{P(H_0)}$

Λ	0	0.33	1.5	6

$P(X\|H)$	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	0	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

0.2

0.8

$P(H, X)$	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	0	0.02	0.06	0.12
H_0	0.32	0.24	0.16	0.08