Last lecture

Maximum Likelihood Estimation (MLE) (Ch 2.8)

- Definition
- Motivation and Examples
- Method

Markov and Chebychev inequalities (Ch 2.9)

- Markov inequality
- Chebychev inequality

Agenda

Markov and Chebychev inequalities (Ch 2.9)

- Chebychev inequality
- Confidence interval

Binary Hypothesis Testing (Ch 2.11)

- Definition
- Likelihood table
- Maximum likelihood decision rule

Chebychev Inequality

Give information regarding $Var(X)$

If X is a RV, for $\mathrm{d} > 0$

- $P\{|X - \mu_X| \geq d\} \leq \frac{\sigma_X^2}{d^2}$

- $P\{|X - \mu_X| \geq a\sigma_X\} \leq \frac{1}{a^2}$

- Proof - Extension of Markov inequality

Confidence Interval

How close is our estimate $\hat{p}$ to the real parameter p

- Do a poll of 200 people - X denotes # of people agree
- $X \sim Bi(n = 200, p)$
- $\hat{p} =$

- $P\{|X - np| \geq a\sigma\} \leq \frac{1}{a^2}$
- $P\left\{\left|\frac{X}{n} - p\right| \leq \frac{a\sigma}{n}\right\} \geq 1 - \frac{1}{a^2}$

- $(\hat{p} - a\sqrt{\frac{p(1-p)}{n}}, \hat{p} + a\sqrt{\frac{p(1-p)}{n}})$ is called **Confidence interval**

Closer look – confidence interval

- $(\hat{p} - a\sqrt{\frac{p(1-p)}{n}}, \hat{p} + a\sqrt{\frac{p(1-p)}{n}})$ is called **Confidence interval**
- $P\left\{p \in \left(\hat{p} - a\sqrt{\frac{p(1-p)}{n}}, \hat{p} + a\sqrt{\frac{p(1-p)}{n}}\right)\right\} \leq 1 - \frac{1}{a^2}$

- Before starting the poll, if we take $a = 5$,
  - we have $1 - \frac{1}{25} = 96\%$ confidence that p will locate at this interval
  - But we don't know $p(1-p)$? Replace it with a loose bound $p(1-p) <$
  - $P\left\{p \in \left(\hat{p} - \frac{a}{2\sqrt{n}}, \hat{p} + \frac{a}{2\sqrt{n}}\right)\right\} \leq 1 - \frac{1}{a^2}$

Example

We want to do an opinion poll of size n for a policy.

- X is # of positive votes
- $\hat{p} = \frac{X}{n}$ be the estimate of the support rate.
- If we want true p within 0.1 with 96% confidence
- How many participants n do we need?

#1 Midterm Reminder

- Oct 6 (Mon.) 7-8:30 PM @ 1002/1013/1015 ECEB
  - Conflict - Oct 7 8-9:30AM @ 2015 ECEB
- All topics until confidence intervals (HERE)
- 1 Letter size HAND-WRITTEN notes
- No calculator
- Exam will be scanned and graded on Gradescope

Hypothesis Testing

Binary Hypothesis Testing with Discreet Observations

Given two hypotheses H_1 and H_0 where $H_0 = H_1^C$

- E.g., $H_1 \triangleq$ “Patient has tumor(s)”; $H_0 \triangleq$ “Patient has no tumor”
- Decision rule
  - Decide H_1 or H_2 given X
  - E.g., $X \triangleq$”Suspect circles in ultrasound scan”
  - How can we pick the best rule?

Maximum Likelihood Table

- Table showing   of two hypotheses

	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	0	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

- Decision rule can be shown on the table

False alarm and missing

	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	0	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

- $P_{false\ alarm} =$
- $P_{miss} =$

	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	0	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

Maximum Likelihood (ML) decision rule

Pick whichever is higher per column!

	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	0	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

What’s the problem?