Last lecture

Maximum Likelihood Estimation (MLE) (Ch 2.8)

- Definition
- Motivation and Examples
- Method

Markov and Chebychev inequalities (Ch 2.9)

- Markov inequality
- Chebychev inequality

Agenda

Markov and Chebychev inequalities (Ch 2.9)

- Chebychev inequality
- Confidence interval

Midterm Logistic

Binary Hypothesis Testing (Ch 2.11)

- Definition
- Likelihood table
- Maximum likelihood decision rule

Chebychev Inequality

Give information regarding $Var(X)$

If X is a RV, for d > 0

- $P\{|X - \mu_X| \geq d\} \leq \frac{\sigma_X^2}{d^2}$

$d = a\sigma_X$

- $P\{|X - \mu_X| \geq a\sigma_X\} \leq \boxed{\frac{1}{a^2}}$

fix scalar.

- Proof - Extension of Markov inequality

$$P\{Y \geq c\} \leq \frac{E[Y]}{c}$$

$E[(X-\mu_X)^2]$

Let $Y = (X - \mu_X)^2$

$$P\{(X-\mu_X)^2 \geq \cancel{c}\} \leq \frac{\sigma_X^2}{\cancel{c}}$$

d^2 $\qquad$ d^2

sqrt.

Confidence Interval

How close is our estimate $\hat{p}$ to the real parameter p

- Do a poll of 200 people - X denotes # of people agree
- $X \sim Bi(n = 200, p)$
- $\hat{p} = \frac{X}{n} = MLE(X, n)$
- $P\{|X - \underbrace{np}_{\mu_x}| \geq a\sigma\} \leq \frac{1}{a^2}$ $\quad \div n$
- $P\left\{\left|\underbrace{\frac{X}{n}}_{\hat{p}} - p\right| \leq \frac{a\sigma}{n}\right\} \geq 1 - \frac{1}{a^2}$
- $\left(\hat{p} - a\sqrt{\frac{p(1-p)}{n}}, \hat{p} + a\sqrt{\frac{p(1-p)}{n}}\right)$ is called **Confidence interval**

$\frac{a\sigma}{n}$

$\sigma_x^2 = np(1-p)$

$\sigma_x = \sqrt{np(1-p)}$

Closer look – confidence interval

- $\left(\hat{p} - a\sqrt{\frac{p(1-p)}{n}}, \hat{p} + a\sqrt{\frac{p(1-p)}{n}}\right)$ is called **Confidence interval**
- $P\left\{p \in \left(\hat{p} - a\sqrt{\frac{p(1-p)}{n}}, \hat{p} + a\sqrt{\frac{p(1-p)}{n}}\right)\right\} \cancel{\leq} \geq 1 - \frac{1}{a^2}$

$1 - \frac{1}{25} = 96\%$

- Before starting the poll, if we take $a = 5$,
  - we have $1 - \frac{1}{25} = 96\%$ confidence that p will locate at this interval
  - But we don't know $p(1-p)$? Replace it with a loose bound $p(1-p) < \frac{1}{4}$ when $p = 0.5$
  - $P\left\{p \in \left(\hat{p} - \frac{a}{2\sqrt{n}}, \hat{p} + \frac{a}{2\sqrt{n}}\right)\right\} \cancel{\leq} \geq 1 - \frac{1}{a^2}$

$\frac{5}{2\sqrt{n}}$

Example

We want to do an opinion poll of size n for a policy.

- X is # of positive votes
- $\hat{p} = \frac{X}{n}$ be the estimate of the support rate. 10%
- If we want true p within 0.1 with 96% confidence
- How many participants n do we need? 625

if $\hat{p} = 37.6\%$

$p \in [27.6\%, 47.6\%]$ with 96%

$\hat{p} \pm \boxed{\frac{\cancel{a}\ 5}{2\sqrt{n}}} \geq 1 - \frac{1}{\cancel{a^2}\ 25}$ 0.96

$1 - \frac{1}{a^2} = 0.96$

$\frac{1}{a^2} = 0.04$

$a = 5$

$n \geq 625$

$\frac{5}{2\sqrt{n}} \leq 0.1$

$\sqrt{n} \geq 25$

Example

We want to do guess the phone busy rate p from survey size n

- X is # of busy lines
- $\hat{p} = \frac{X}{n}$ be the estimate of the support rate.
- If we want true p within 0.05 with 99% confidence
- How many phones n do we need to check?

$$P\left\{p \in \left(\hat{p} - \frac{a}{2\sqrt{n}}, \hat{p} + \frac{a}{2\sqrt{n}}\right)\right\} \geq 1 - \frac{1}{a^2}$$

99%

#4151495

$1 - \frac{1}{a^2} = 99\%$

$a = 10$

$\frac{10}{2\sqrt{n}} \leq 0.05$

$\sqrt{n} \geq 100.$

$n \geq 10000$

#1 Midterm Reminder

- Oct 6 (Mon.) 7-8:30 PM @ 1002/1013/1015 ECEB
  - Conflict - Oct 7 8-9:30AM @ 2015 ECEB
- All topics until confidence intervals (HERE)
- 1 Letter size HAND-WRITTEN notes (2-sided)
- No calculator
- Exam will be scanned and graded on Gradescope

2 Midterm ↓

Hypothesis Testing

Binary Hypothesis Testing with Discreet Observations

Given two hypotheses H_1 and H_0 where $H_0 = H_1^C$

- E.g., $H_1 \triangleq$ "Patient has tumor(s)"; $H_0 \triangleq$ "Patient has no tumor"
- Decision rule
  - Decide H_1 or H_2 given X → suspicious
  - E.g., $X \triangleq$"Suspect circles in ultrasound scan"
  - How can we pick the best rule?

Maximum Likelihood Table

- Table showing likelihood of two hypotheses

		$X=0$	$X=1$	$X=2$	$X=3$	
$P(X \mid H_1) \rightarrow$	H_1	0	$\underline{0.1}$	$\underline{0.3}$	$\underline{0.6}$	$\Sigma = 1$
$P(X \mid H_2) \rightarrow$	H_0	$\underline{0.4}$	0.3	0.2	0.1	$\Sigma = 1$

- Decision rule can be shown on the table

underscore choice per column