Last lecture

Maximum Likelihood Estimation (MLE) (Ch 2.8)

- Definition
- Motivation and Examples
- Method

Markov and Chebychev inequalities (Ch 2.9)

- Markov inequality
- Chebychev inequality

Agenda

Markov and Chebychev inequalities (Ch 2.9)

- Chebychev inequality
- Confidence interval
- Hidtern Logistic.
 Binary Hypothesis Testing (Ch 2.11)

- **Definition**
- Likelihood table
- Maximum likelihood decision rule

Chebychev Inequality

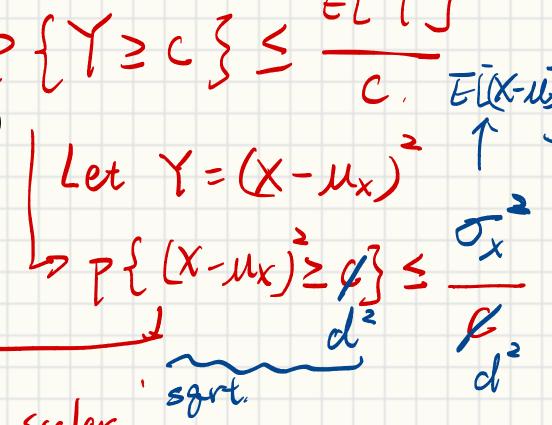
Give information regarding Var(X)

If X is a RV, for d > 0

$$P\{|X - \mu_X| \ge d\} \le \frac{\sigma_X^2}{d^2}$$

•
$$P\{|X - \mu_X| \ge a\sigma_X\} \le \frac{1}{a^2}$$

Proof - Extension of Markov inequality



Confidence Interval

How close is our estimate \hat{p} to the real parameter p

- Do a poll of 200 people X denotes # of people agree
- $X \sim Bi(n = 200, p)$

•
$$\hat{p} = \frac{X}{n} = MLE(X, n)$$

•
$$P\{|X - np| \ge a\sigma\} \le \frac{1}{a^2}$$

$$(|X - np| \ge a\sigma) \le \frac{1}{a^2}$$

•
$$\hat{p} = \frac{X}{X} = \text{MLE}(X, n)$$

• $P\{|X - np| \ge a\sigma\} \le \frac{1}{a^2}$
• $P\{\left|\frac{X}{n} - p\right| \le \frac{a\sigma}{n}\} \ge 1 - \frac{1}{a^2}$
• $P\{\left|\frac{X}{n} - p\right| \le \frac{a\sigma}{n}\} \ge 1 - \frac{1}{a^2}$

•
$$(\hat{p} - a\sqrt{\frac{p(1-p)}{n}}, \hat{p} + a\sqrt{\frac{p(1-p)}{n}})$$
 is called **Confidence interval**

Closer look – confidence interval

•
$$(\hat{p} - a\sqrt{\frac{p(1-p)}{n}}, \hat{p} + a\sqrt{\frac{p(1-p)}{n}})$$
 is called **Confidence interval**

•
$$P\left\{p \in \left(\hat{p} - a\sqrt{\frac{p(1-p)}{n}}, \hat{p} + a\sqrt{\frac{p(1-p)}{n}}\right)\right\} \ge 1 - \frac{1}{a^2}$$

- Before starting the poll, if we take a = 5,
 - we have $1 \frac{1}{25} = 96\%$ confidence that p will locate at this interval
 - But we don't know p(1-p)? Replace it with a loose bound $p(1-p) < \frac{1}{4}$ when p = 0.5

$$p(1-p) < \frac{1}{4} \text{ then } p = 0.5$$

$$P\left\{p \in \left(\hat{p} - \left(\frac{a}{2\sqrt{n}}\right), \hat{p} + \frac{a}{2\sqrt{n}}\right)\right\} \not\leq 1 - \frac{1}{a^2}$$

Example

We want to do an opinion poll of size n for a policy.

- X is # of positive votes
- $\hat{p} = \frac{X}{n}$ be the estimate of the support rate.
- If we want true p within 0.1 with 96% confidence
- How many participants n do we need? 625

if
$$\hat{p} = 37.6\%$$
 $p \in [27.6\%, 47.6\%]$ with 96%
 $1 - \frac{1}{a^2} = 0.96$
 $p \neq \sqrt[4]{3}$
 $2 - \frac{1}{\sqrt{3}} = 0.04$
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Example

We want to do guess the phone busy rate p from survey size n

- X is # of busy lines
- $\hat{p} = \frac{X}{n}$ be the estimate of the support rate.
- If we want true *p* within 0.05 with 99% confidence
- How many phones n do we need to check?

$$P\left\{p \in \left(\hat{p} - \frac{a}{2\sqrt{n}}, \hat{p} + \frac{a}{2\sqrt{n}}\right)\right\} \ge 1 - \frac{1}{a^2} \qquad \text{In}$$



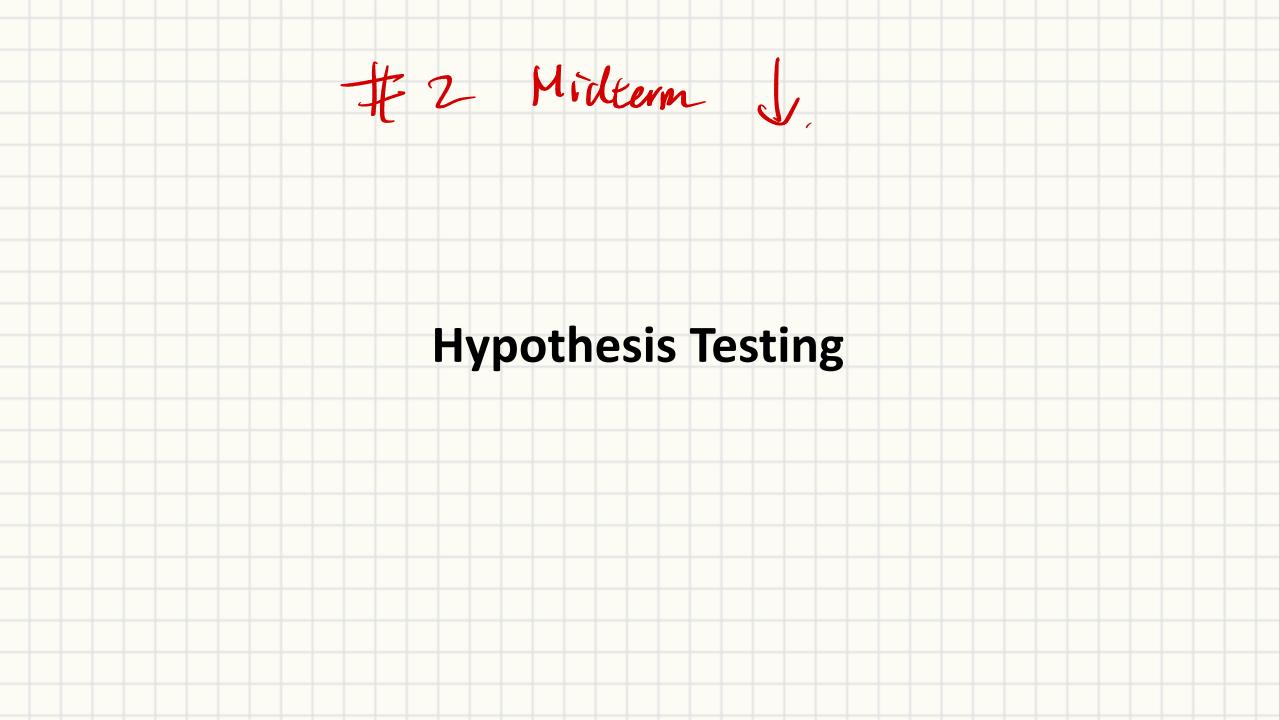
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$$1-\frac{1}{\alpha^2} = 99\%$$

$$a = 10$$

#1 Midterm Reminder

- Oct 6 (Mon.) 7-8:30 PM @ 1002/1013/1015 ECEB
 - Conflict Oct 7 8-9:30AM @ 2015 ECEB
- All topics until confidence intervals (HERE)
- 1 Letter size HAND-WRITTEN notes (2 sided)
- No calculator
- Exam will be scanned and graded on Gradescope



Binary Hypothesis Testing with Discreet Observations

- Given two hypotheses H_1 and H_0 where $H_0 = H_1^C$ E.g., $H_1 \triangleq$ "Patient has tumor(s)"; $H_0 \triangleq$ "Patient has no
 - Decision rule
 - Decide H_1 or H_2 given X suspicions E.g., $X \triangleq$ "Suspect circles in ultrasound scan"

 - How can we pick the best rule?

Maximum Likelihood Table

Table showing likelihood of two hypotheses

$$P(X | H_1)$$
 $X = 0$ $X = 1$ $X = 2$ $X = 3$ H_1 0 0.1 0.3 0.6 $P(X | H_2) > H_0$ 0.4 0.3 0.2 0.1

Decision rule can be shown on the table

under score choice per column