Last lecture

Example for binomial distribution

Geometric distribution (Ch 2.5)

- Definition
- Examples
- Property – memoryless

$p_G(k) =$

$\mu_G =$

$Var(G) = \sigma_G^2 =$

Agenda

Geometric distribution (Ch 2.5)

- Property – memoryless

Bernoulli Process (Ch 2.6)

- Definition
- Properties
- Negative binomial distribution

Poisson Process (Ch 2.6)

- Definition

Property – Memoryless property

For geometric series, failing 10 times will not affect the 11-th trial

- $P\{L > k + n | L > n\} =$

- Called “memoryless property”
- What’s the expected total number to get the first 1 after getting {0,0,0,0}?

Game – Push the luck (simplified Incan-Gold)

Start a game with infinite rounds and 0 points

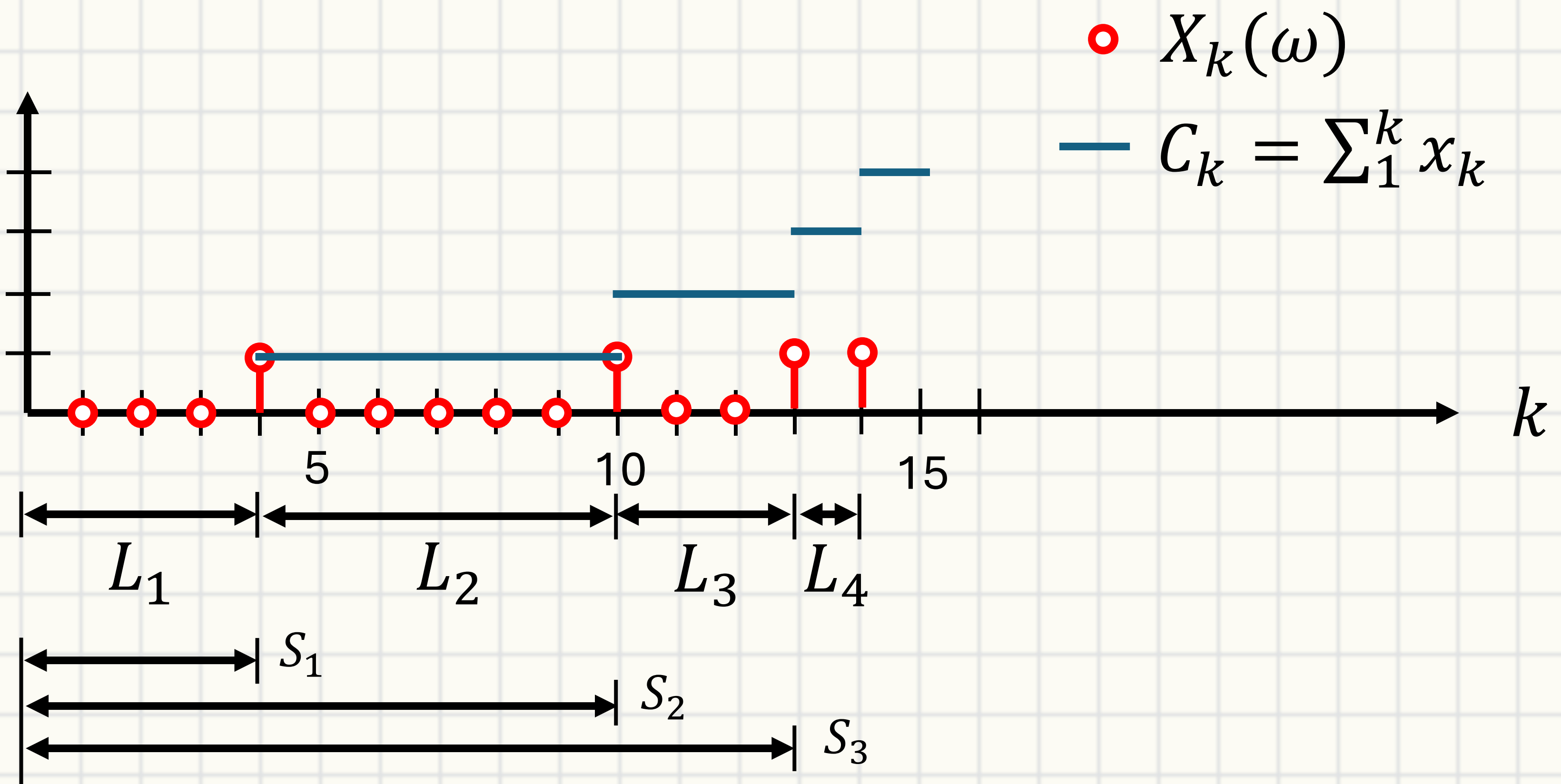
- 2 Actions per round – Go or Keep
- Go
  - $p = \frac{2}{3}$ win 1 point
  - Otherwise, lose all points
- Keep
  - Deposit the current point and end the game
- What's the best strategy?

Bernoulli Process

Bernoulli Process Definition

An infinite sequence $X_1, X_2 \ldots$ s.t. $X_k \sim Bern(p)$

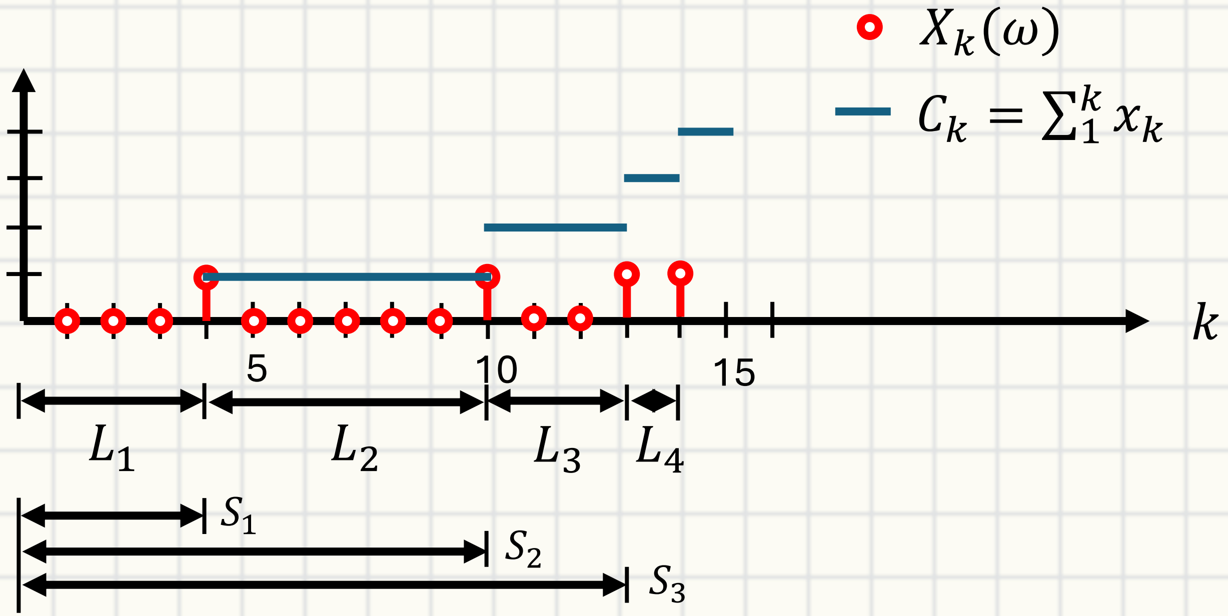
- ω is a possible outcome of the sequence
- $X_k(\omega)$ is called a "   " of outcome ω



Bernoulli Process Definition

Observe that a Bernoulli process can be defined by

1. $X_k \sim Bern(p)$
2. $C_k \sim B(k, p)$
3. $L_k \sim L(p)$
4. $S_r = \sum_1^r L_r$: # of trials required to get r ones

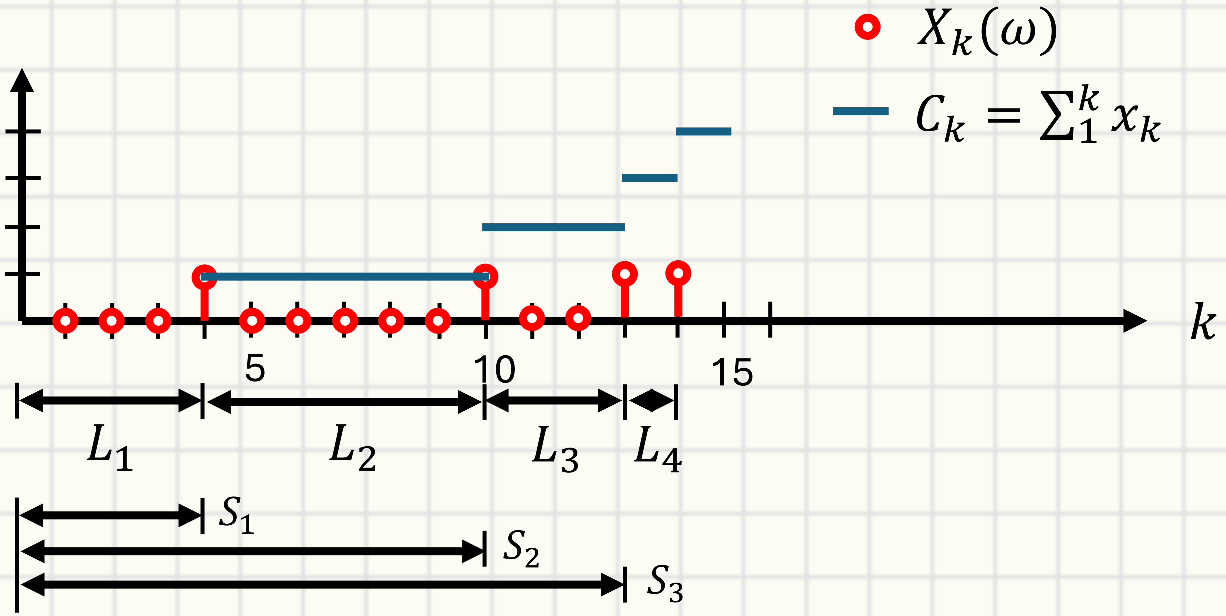


S_r - Negative Binomial Distribution

What is the pmf of S_r with parameter (r, p)?

- # of trials required to get r ones

- $p_S(n) =$



Poisson Distribution

Poisson Distribution $\boldsymbol{Pois(\lambda)}$

A binomial distribution with large n, small p, and $\lambda = np$

- Example – Misspelled words in a document
  - Many number of words n
  - Small misspelled rate p

- When we care about the "rate" np
  - We only have the mean np
  - We know p is small

- $p_X(k) =$

Poisson Distribution $Pois(\lambda)$

- Why $p_X(k) = \frac{e^{-\lambda}\lambda^k}{k!}$?
  - $p_X(k) \propto \frac{\lambda^k}{k!}$
  - $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$
  - $\mu_x =$
  - $\sigma_x^2 =$

Poisson Distribution Example

A coffee shop has in average 6 customers per hour, what’s the probability that there are NO customers in the next hour?