Last lecture

Example for binomial distribution

Geometric distribution (Ch 2.5)

- Definition
- Examples
- Property – memoryless

$$p_G(k) = (1-p)^{k-1}p.$$

$$\mu_G = \frac{1}{p}.$$

$$Var(G) = \sigma_G^2 = \frac{(1-p)}{p^2}$$

Agenda

Geometric distribution (Ch 2.5)

- Property – memoryless

Bernoulli Process (Ch 2.6)

- Definition
- Properties
- Negative binomial distribution

Poisson Process (Ch 2.6)

- Definition

Property – Memoryless property

For geometric series, failing 10 times will not affect the 11-th trial

Get k more tails

- $P\{L > k + n | L > n\} =$ $P\{L > k\}$

Get n tails.

- Called "memoryless property"
- What's the expected total number to get the first 1 after getting {0,0,0,0}?

$E[L] + 4 = \frac{1}{p} + 4.$

$E[L] = \sum_k P(L = k + \cancel{n} | L > \cancel{n}) = P(L = k)$

4 4

Game – Push the luck (simplified Incan-Gold)

Start a game with infinite rounds and 0 points

- 2 Actions per round – Go or Keep
- Go
  - $p = \frac{2}{3}$ win 1 point
  - Otherwise, lose all points
- Keep
  - Deposit the current point and end the game
- What's the best strategy?

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At start of round k

Backpack $k-1$.

Always Go.

$$\text{Go.} = \frac{2}{3} \boxed{E[I]} \qquad \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n = \frac{\frac{2}{3}}{1-\frac{2}{3}} = 2$$

$$\frac{1}{3} \quad 0.$$

$$\text{Keep.} \quad V = \underline{k-1}$$

$k-1 > 2$ Keep $k > 3$

Bernoulli Process

Bernoulli Process Definition

An infinite sequence $X_1, X_2 \ldots \to X_\infty$ s.t. $X_k \sim Bern(p)$

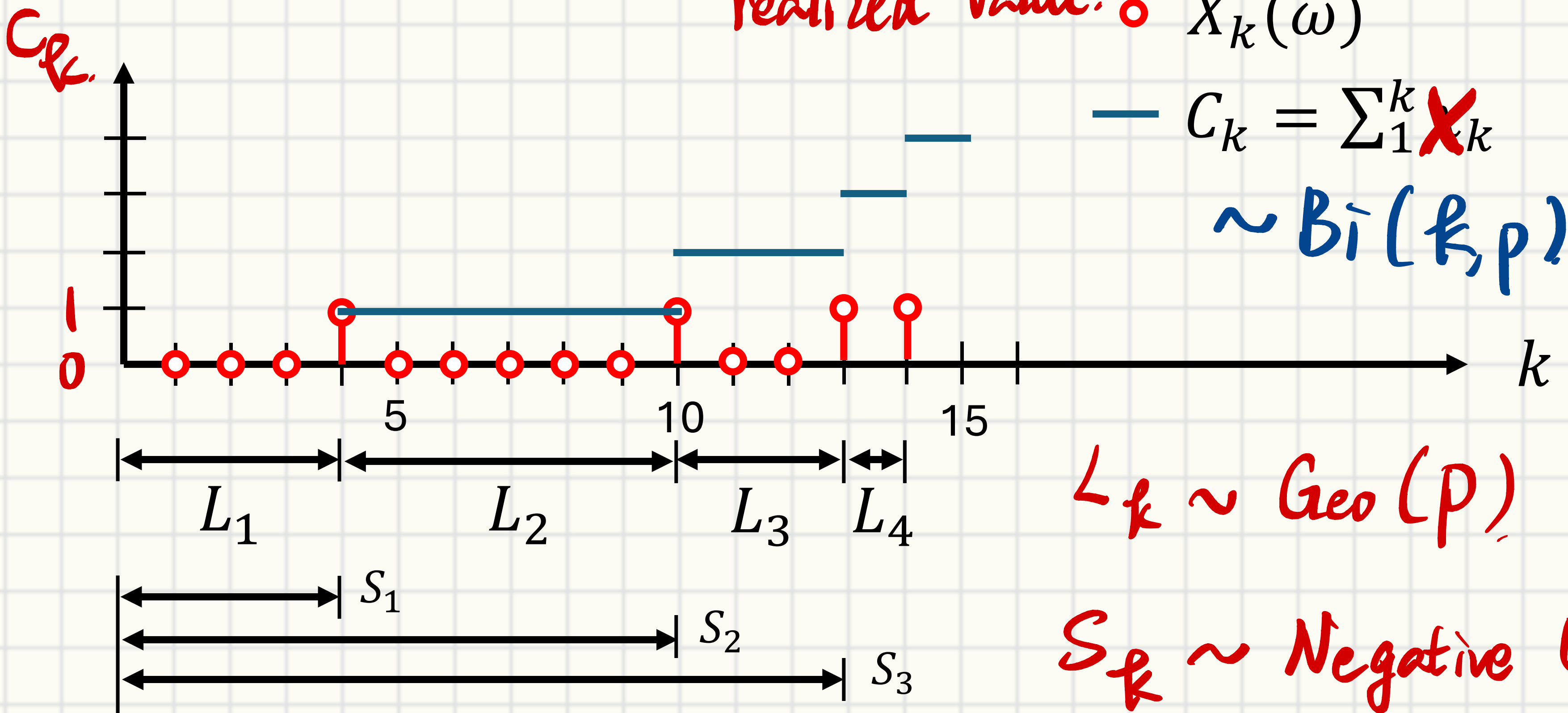
- ω is a possible outcome of the sequence
- $X_k(\omega)$ is called a " realization " of outcome ω

realized value.

○ $X_k(\omega)$

— $C_k = \sum_1^k \cancel{X}_k$

$\sim Bi(k, p)$



$S_1 \sim Geo$

$k=1$

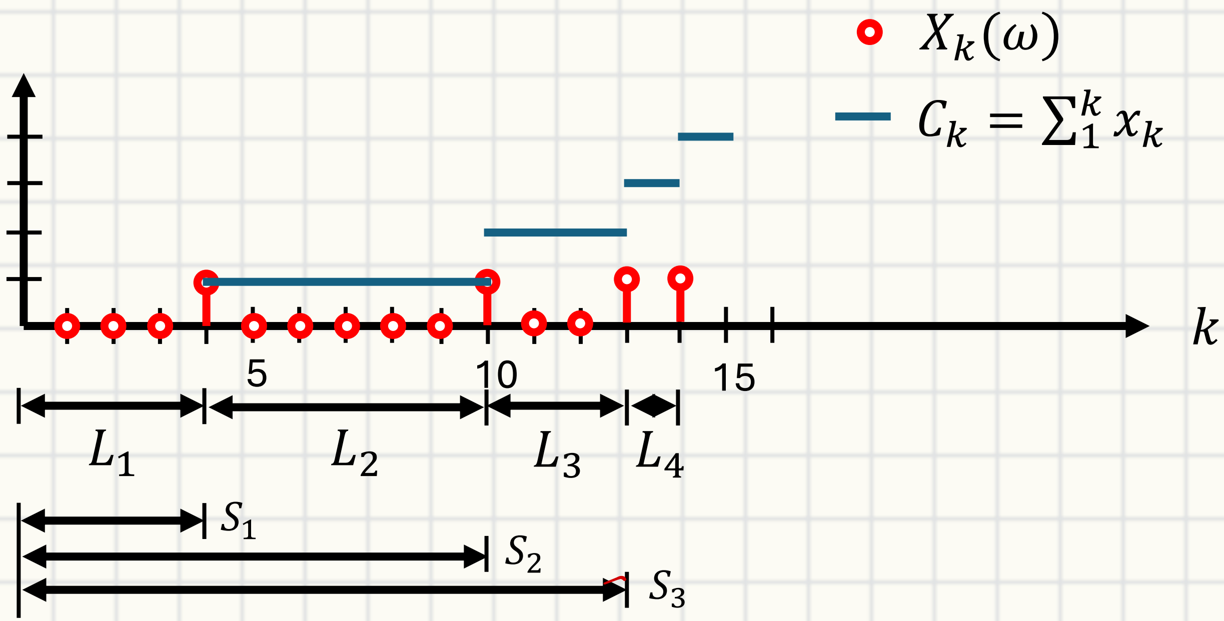
$L_k \sim Geo(p)$

$S_k \sim$ Negative Binomial(k, p)

Bernoulli Process Definition

Observe that a Bernoulli process can be defined by

1. $X_k \sim Bern(p)$
2. $C_k \sim Bi(k, p)$
3. $L_k \sim G(p)$
4. $S_r = \sum_1^r L_r$: # of trials required to get r ones



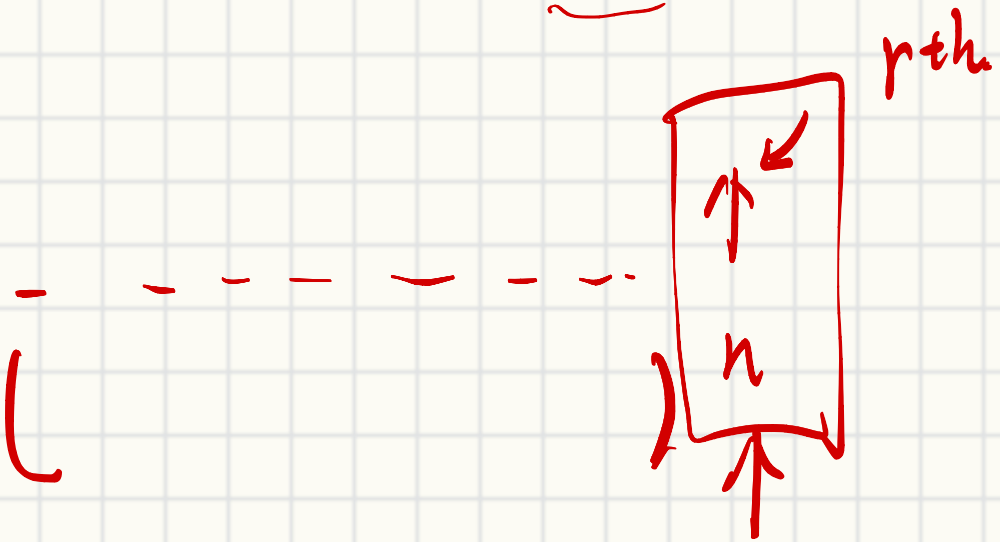
S_r - Negative Binomial Distribution

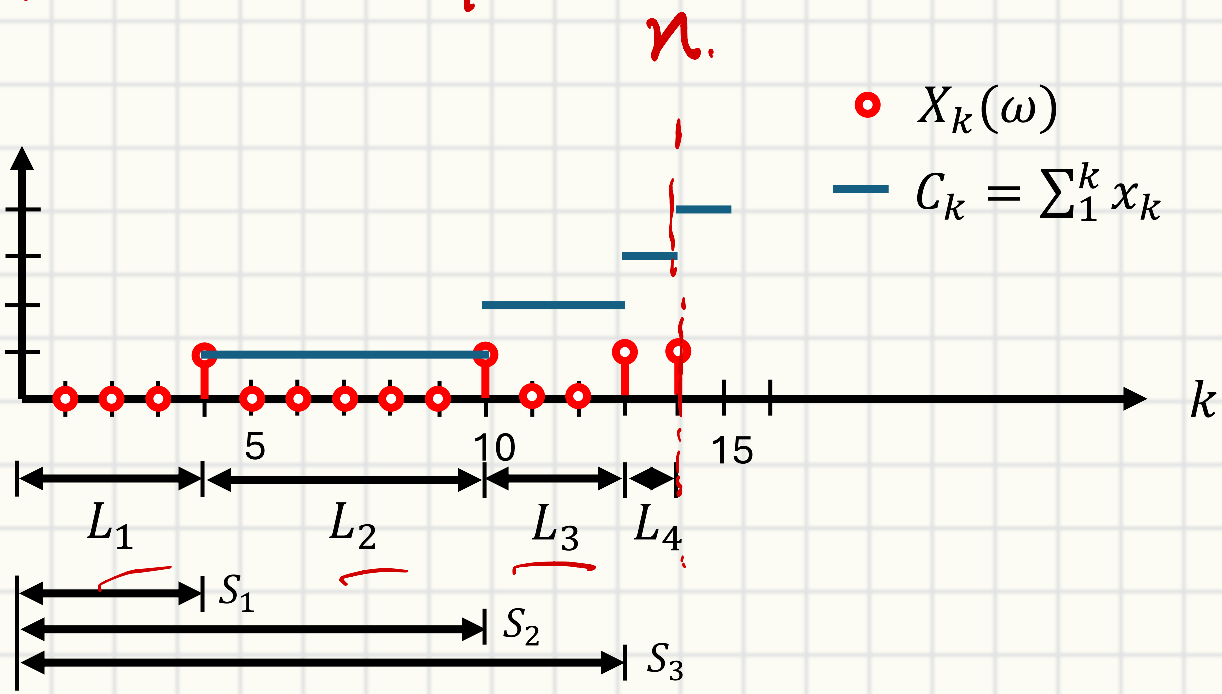
$\mu_S = r\,\mu_G = \frac{r}{p}$

$\sigma_S^2 = r\,\sigma_S^2 = \frac{r(1-p)}{p^2}$

What is the pmf of S_r with parameter (r, p)?

- # of trials required to get r ones
- $p_S(n) = \binom{n-1}{r-1} p^{(r-1)} (1-p)^{[n-1-(r-1)]} p. = \binom{n-1}{r-1} p^r (1-p)^{n-r}$





Poisson Distribution

Poisson Distribution $\boldsymbol{Pois}(\lambda)$

$\mu_{Bi} = np$

A binomial distribution with large n, small p, and $\lambda = np$

- Example – Misspelled words in a document
  - Many number of words n
  - Small misspelled rate p

$p \downarrow \quad n \uparrow$

$Pois(\lambda) \to Bi(n, p)$

- When we care about the "rate" np
  - We only have the mean np
  - We know p is small
- $p_X(k) = \dfrac{\lambda^k e^{-\lambda}}{k!}$

Poisson Distribution $\boldsymbol{Pois}(\lambda)$

- Why $p_X(k) = \frac{e^{-\lambda}\lambda^k}{k!}$?
  - $p_X(k) \propto \frac{\lambda^k}{k!}$

    k hend.
  - $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$
  - $\mu_x =$ $np = \lambda$
  - $\sigma_x^2 = \lambda.$

σ_B^2

$np(1-p)$

Poisson Distribution Example

A coffee shop has in average 6 customers per hour, what's the probability that there are NO customers in the next hour?

30mins

$$\lambda = 6$$

$$P_x(0) = \frac{e^{-6} 6^0}{0!} = e^{-6}$$

$$\lambda = \frac{6}{2} = 3$$

$$P_x(0) = \frac{e^{-3} 3^0}{0!} = e^{-3}$$