Last lecture

Independent Events/ RVs (Ch 2.4)

- Examples and Facts

Distributions (Ch 2.4)

- Bernoulli
- Binomial

$$p_X(1) = p$$

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Agenda

Example for binomial distribution

Geometric distribution (Ch 2.5)

- Definition
- Examples
- Property – memoryless

Bernoulli Process (Ch 2.6)

- Definition
- Properties
- Negative binomial distribution

Binomial Example – Best of K

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Team A and B play "Best of 7" games

- No tie, whoever wins 4 games out of 7 is the match winner
- E.g. $w_i = \{A, A, A, B, A\}$: the winner is A
- Let p denotes A's win rate per game
- Y denotes the number of games played, $p_Y(k) = ?$

Geometric Distributions

Geometric Distribution

of Toss on a (unfair) coin until the first Head is shown

Conduct independent Bernoulli trials of parameter p

- $L \triangleq$ # of trials until we get the first 1
- $p_L(1) =$
- $p_L(2) =$
- $p_L(k) =$
- $P\{L > k\} =$

Mean

$$E[L] = 1 \times p + (1 - p) \times (E[L] + 1)$$

Variance

$$Var(L) = E[L^2] - \mu_L^2$$

Example

What's the expected number of rolls to get 1 to 6 at least once?

- Example series :
  $\{2, 4, 2, 3, 4, 4, 3, 5, 3, 5, 4, 4, 6, 2, 3, 3, 4, 1\}$

- $R_k \triangleq$ # roll to get the k-th unseen number

Property – Memoryless property

For geometric series, failing 10 times will not affect the 11-th trial

- $P\{L > k + n | L > n\} =$

- Called "memoryless property"
- What's the expected total number to get the first 1 after getting {0,0,0,0}?

Game – Push the luck (simplified Incan-Gold)

Start a game with infinite rounds and 0 points

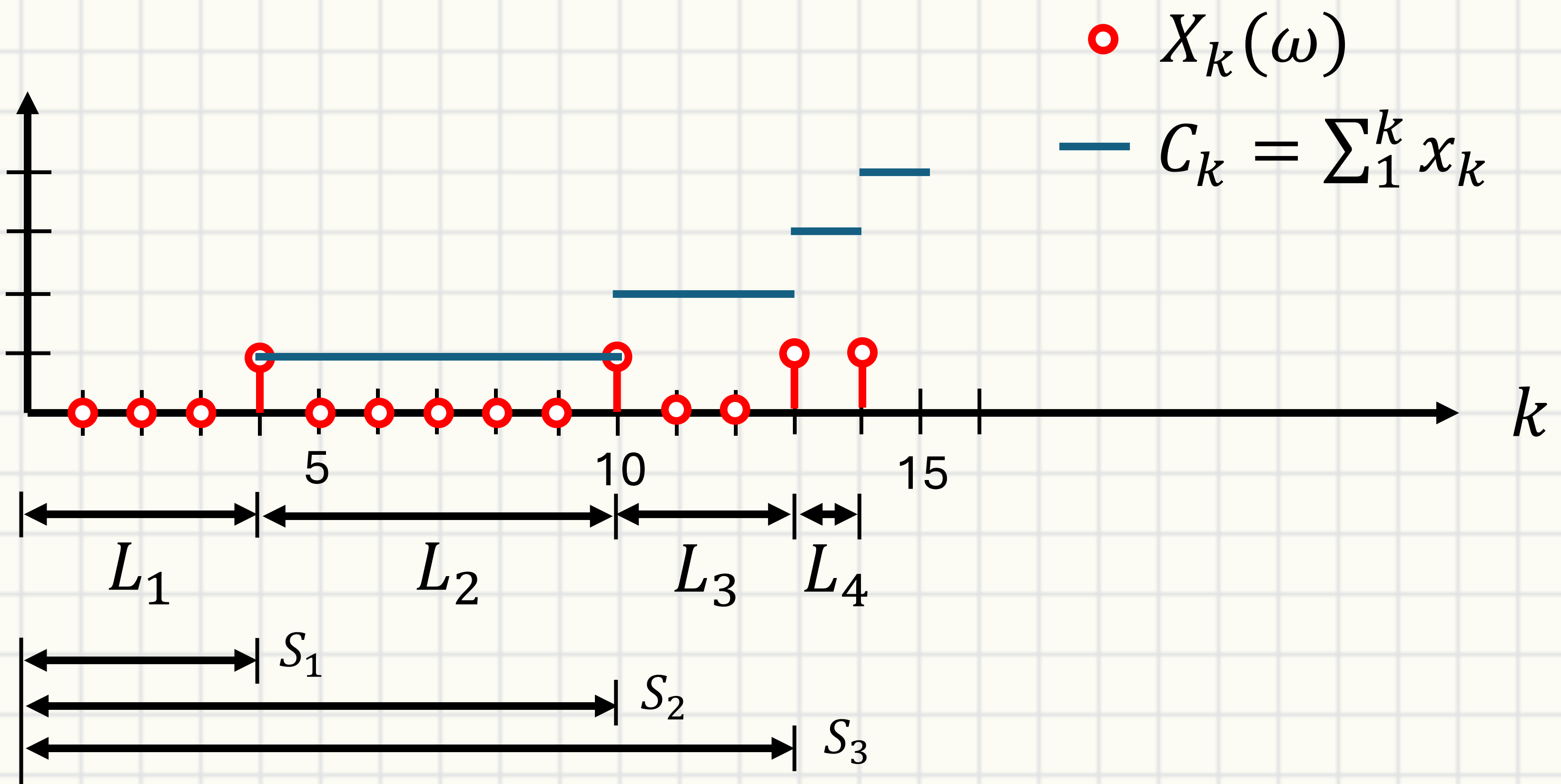
- 2 Actions per round – Go or Keep
- Go
  - $p = \frac{2}{3}$ win 1 point
  - Otherwise, lose all points
- Keep
  - Deposit the current point and end the game
- What's the best strategy?

Bernoulli Process

Bernoulli Process Definition

An infinite sequence X_1, X_2 … s.t. $X_k \sim Bern(p)$

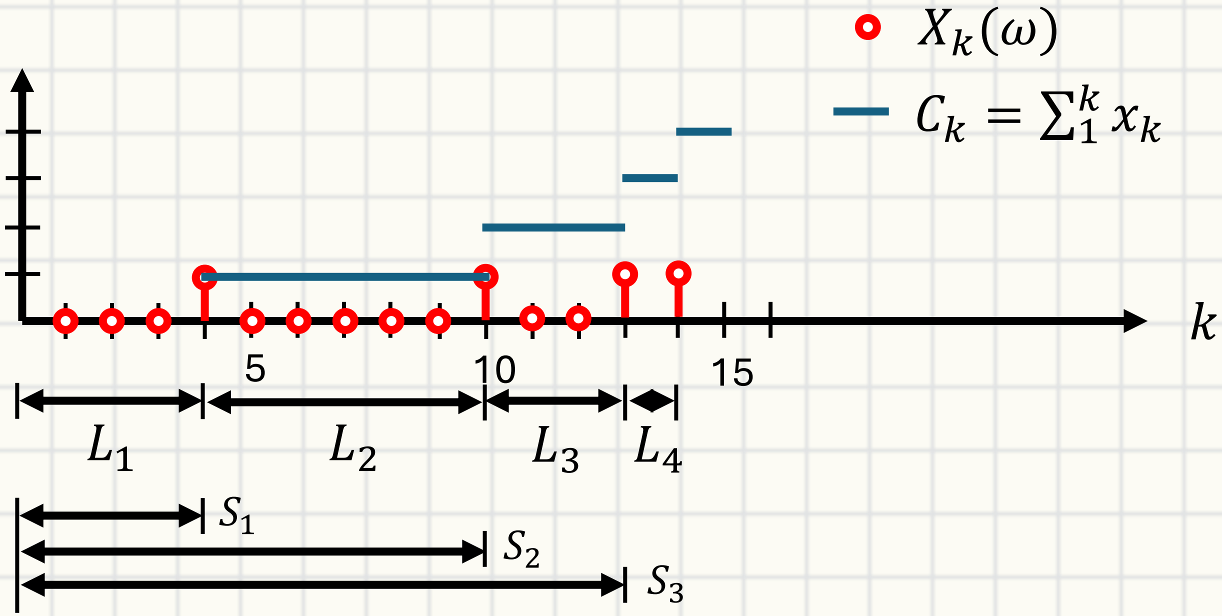
- ω is a possible outcome of the sequence
- $X_k(\omega)$ is called a “   ” of outcome ω



Bernoulli Process Definition

Observe that a Bernoulli process can be defined by

1. $X_k \sim Bern(p)$
2. $C_k \sim B(k, p)$
3. $L_k \sim L(p)$
4. $S_r = \sum_1^r L_r$: # of trials required to get r ones



S_r - Negative Binomial Distribution

What is the pmf of S_r with parameter (r, p)?

- # of trials required to get r ones

- $p_S(n) =$