Last lecture

Bayes formula (Ch 2.10)

- Examples

Independent Events/ RVs (Ch 2.4)

- Definition
- Motivation
- Examples and Facts

$P(B|A) =$

$P(AB) =$

Agenda

Independent Events/ RVs (Ch 2.4)

- Examples and Facts

Distributions (Ch 2.4)

- Bernoulli
- Binomial

Geometric distribution (Ch 2.5)

- Definition

Terms and Facts

- A, B, C are pairwise independent if $(A, B), (B, C), (A, C)$ are mutually independent
  - Toss a fair coin twice
  - $A \triangleq$ {First coin is Head}
  - $B \triangleq$ {second coin is Head}
  - $C \triangleq$ {toss results are the same}

Terms and Facts

- A, B, C are independent they are pairwise independent and $P(ABC) = P(A)P(B)P(C)$

- $A_1, A_2, \dots A_i$ are independent if

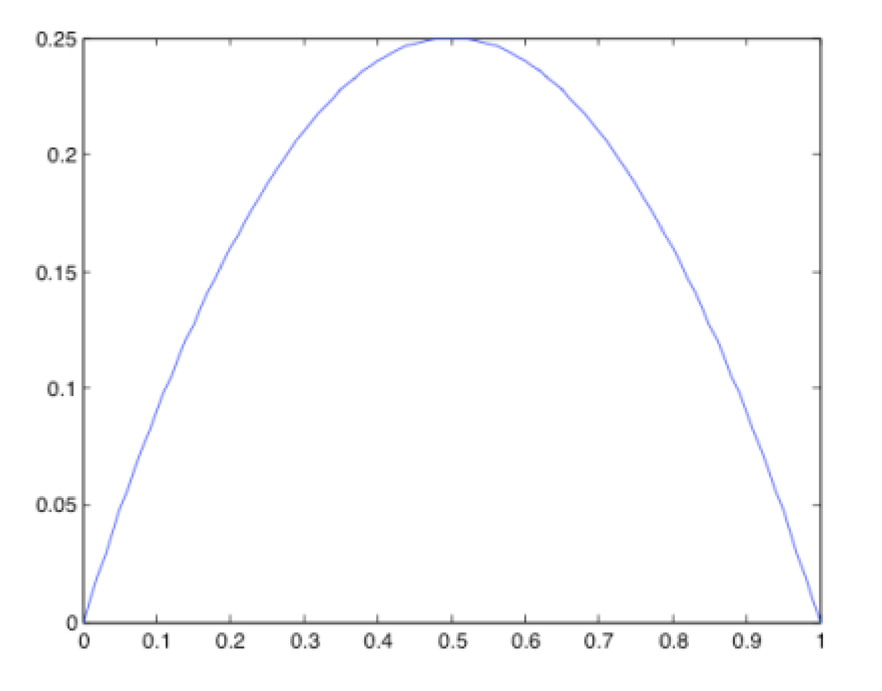
$$P\left(A_{i_1} A_{i_2} \dots A_{i_k}\right) = P\left(A_{i_1}\right) P\left(A_{i_2}\right) \dots P(A_{i_k})$$

Distributions

Bernoulli Distribution

X is Bernoulli distribution with parameter p if

- $P\{X = 1\} = p$ and $P\{X = 0\} = 1 - p$
- "Toss a (unfair) coin with p probability Head"
- Only possible outcomes, pmf contains bins
- $E[X] =$
- $E[X^2] =$
- $\sigma_x^2 =$

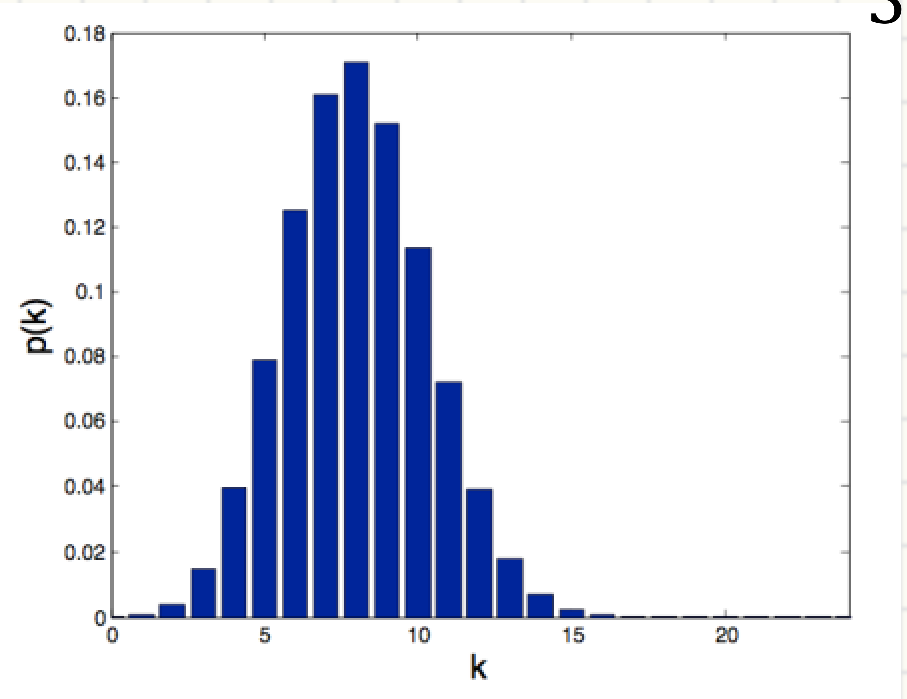


Binomial Distribution

X is binomial distribution with parameter (n, p) if

- X is sum of n Bernoulli trials with parameter p
- Draw the unfair coin n times and count the Head

$$(n, p) = (24, \frac{1}{3})$$



- $p_X(k) =$

Binomial Distribution

$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$

Proof - $\sum_{k=0}^{n} p_X(k) = 1$

- $(1+x)^n = \sum_{k=0}^{n} x^k$
- $x = \frac{p}{1-p}$

Binomial mean

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$E[X] = \sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k}$$

$$= \sum_{k=1}^{n} k \frac{n!}{(n-k)!k!} p^k (1-p)^{n-k}$$

$$= np \sum_{k=1}^{n} \frac{(n-1)!}{(n-k)!(k-1)!} p^{k-1} (1-p)^{n-k}$$

$$= np \sum_{m=0}^{n-1} \frac{n-1!}{(n-1-m)!m!} p^m (1-p)^{n-1-m}$$

$$= np \sum_{m=0}^{j} \binom{j}{m} p^m (1-p)^{j-m}$$

Binomial Properties

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

- Mean $E[X] =$
- Variance $Var(X) =$
  - why?

- Shape of the pmf- what is the most likely k?
  - $k^* = \lfloor (n+1)p \rfloor$

Binomial Example – Best of K

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Team A and B play "Best of 7" games

- No tie, whoever wins 4 games out of 7 is the match winner
- E.g. $w_i = \{A, A, A, B, A\}$: the winner is A
- Let p denotes A's win rate per game
- Y denotes the number of games played, $p_Y(k) = ?$

Geometric Distributions

Geometric Distribution

of Toss on a (unfair) coin until the first Head is shown

Conduct independent Bernoulli trials of parameter p

- $L \triangleq$ # of trials until we get the first 1
- $p_L(1) =$
- $p_L(2) =$
- $p_L(k) =$
- $P\{L > k\} =$

Geometric Distribution

$$E[L] = 1 \times p + (1 - p) \times (E[L] + 1)$$

Geometric Distribution

$$Var(L) =$$