LECTURE 7 : THE VARIANCE OF A DISCRETE-TYPE RANDOM VARIABLE

• TOPICS TO COVER (BASED ON CH 2.2)

A MEASURE OF DISPERSION / SPREAD

→ THE VARIANCE OF A DISCRETE-TYPE RANDOM VARIABLE

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→ THE VARIANCE OF DISCRETE-TYPE A RANDOM VARIABLE

RE ⇝ $(\Omega, \mathcal{F}, P)$ $\quad P: \mathcal{F} \longrightarrow \mathbb{R}$

$X : \Omega \longrightarrow \mathbb{R}$ $\quad Y : \Omega \longrightarrow \mathbb{R}$

$p_X(u)$: PMF OF X $\quad p_Y(v)$: PMF OF Y

$X = 100$, w.p. 1. $\quad E(X) = 100$

$Y = 100{,}000$, w.p. $\frac{1}{1000}$, $\quad E(Y) = 100$

LOTTERY $\quad 0$, OTHERWISE.

⇒ MEAN DOESN'T TELL ABOUT THE SPREAD

RISK !

SQUARE BECAUSE OF SOME STATISTICAL PROPERTIES

square

DEFINITION: $\mathrm{Var}(X) := E\big(\underbrace{X - \mu_X}\big)^2$

σ_X^2 $\quad$ mean $\quad$ deviation of X from its mean

: MEAN SQUARE DEVIATION OF X FROM μ_X

IF X IS USED AS A PREDICTOR / ESTIMATOR OF μ_X : $X - \mu_X$: ERROR

$\mathrm{Var}(X)$: MEAN SQUARE ERROR (MSE)

ALSO: $$E(\overbrace{X-\mu_X}^{\text{DEV. / ERROR}}) = E(X) - E(\mu_X) \quad \because \text{ LINEARITY}$$

$$= \mu_X - \mu_X$$

$$= 0$$

$$\underbrace{SD(X)}_{\text{STANDARD DEVIATION}} := \sqrt{\sigma_X^2} = \sigma_X \quad : \text{ SAME UNITS AS } X$$

PROPERTIES OF VARIANCE

• DEFINITION: $$Var(X) := E(X-\mu_X)^2 \qquad \cdots (i)$$

$$= E(X^2 + \mu_X^2 - 2X\mu_X)$$

$$= E(X^2) + E(\mu_X^2) - 2\mu_X E(X) \quad \because \text{ LINEARITY}$$

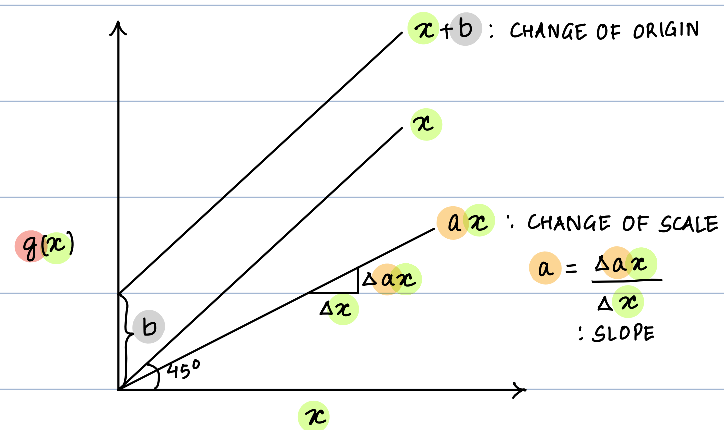
$$= E(X^2) + \mu_X^2 - 2\mu_X^2$$

$$\Rightarrow \quad Var(X) := E(X^2) - \mu_X^2 \qquad \cdots (ii) \quad : \text{ ALTERNATE DEFINITION}$$

• EFFECT OF CHANGE OF ORIGIN AND SCALE

$$Var(\underbrace{a}_{\text{CHANGE OF SCALE}}X + \overbrace{b}^{\text{CHANGE OF ORIGIN}})$$

$$g(X) := aX + b$$



$$Var(aX + b) := E(aX + b)^2 - (E(aX + b))^2 \quad \because \text{ DEF. OF VAR.}$$

LINEARITY

$$= E(a^2X^2 + b^2 + 2abX) - (aE(X) + E(b))^2$$

$$= a^2E(X^2) + b^2 + 2ab\mu_X - (a^2\mu_X^2 + b^2 + 2ab\mu_X)$$

$$= a^2E(X^2) + b^2 + 2ab\mu_X - a^2\mu_X^2 - b^2 - 2ab\mu_X$$

$$= a^2E(X^2) - a^2\mu_X^2$$

$$= a^2(E(X^2) - \mu_X^2)$$

$$= a^2 Var(X)$$ $\Rightarrow$ VARIANCE IS INDEP. OF CHANGE OF ORIGIN BUT NOT OF SCALE

STANDARDIZED RANDOM VARIABLES

$$Z := \frac{X-\mu_X}{\sigma_X}$$

STANDARDIZED VERSION OF X

$$E(Z) = E\left(\frac{X-\mu_X}{\sigma_X}\right) = E\left(\frac{1}{\sigma_X}X - \frac{1}{\sigma_X}\mu_X\right)$$

$$= \frac{1}{\sigma_X}(E(X) - \mu_X)$$

$$= \frac{1}{\sigma_X}(\mu_X - \mu_X) = 0$$

$$Var(Z) = E\left(\frac{X-\mu_X}{\sigma_X}\right)^2 - \left(E\left(\frac{X-\mu_X}{\sigma_X}\right)\right)^2$$

EXERCISE

$$= 1$$

DEFINITION: $E(X^i)$: ith MOMENT OF X

$$Var(X) = \underbrace{E(X^2)}_{\text{2ND MOMENT OF X}} - \underbrace{(E(X))^2}_{\text{SQUARE OF THE 1ST MOMENT OF X}}$$