ECE 313 / MATH 362 PROBABILITY WITH ENGINEERING APPLICATIONS

LECTURE 7: THE MEAN OF A DISCRETE-TYPE RANDOM VARIABLE

- TOPICS TO COVER (BASED ON CH 2.2)

A MEASURE OF CENTRAL TENDENCY

→ THE MEAN OF A DISCRETE-TYPE RANDOM VARIABLE

RE ⇝ $(\Omega, \mathcal{F}, P)$ $P: \mathcal{F} \longrightarrow \mathbb{R}$

$X: \Omega \longrightarrow \mathbb{R}$

$p_X(u)$: PMF

MEAN OF X $:= E(X) := \sum_i u_i \, p_X(u_i)$

EXPECTATION μ_X

- E.G. RE: ROLLING A DIE

X = FACE YOU GET WHEN YOU ROLL A 6-FACED DIE

- PMF OF X: $p_X(u) = P\{X = u\}$

X	$p_X(u) = P\{X=u\}$
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

$$E(X) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + \cdots + 6 \cdot \frac{1}{6} = 3.5$$

E.G. X IS A RV TAKING VALUES FROM $\{-2, -1, 0, 1, 2, 3, 4, 5\}$ EACH WITH PROB. $1/8$.

DEFINE $Y = X^2$ $Y = g(X)$ where $g(u) = u^2$

WE WANT $E(Y) := \sum_{v_i} v_i \, p_Y(v_i)$

PMF OF Y

WE CALCULATE THE PMF OF Y AS FOLLOWS:

$Y = v$	X	$p_Y(v) = P\{Y = v\}$
0	0	$1/8$
1	-1, 1	$2/8$
4	-2, 2	$2/8$
9	3	$1/8$
16	4	$1/8$
25	5	$1/8$

$$E(Y) = 0 \cdot \frac{1}{8} + 1 \cdot \frac{2}{8} + 4 \cdot \frac{2}{8} + 9 \cdot \frac{1}{8} + 16 \cdot \frac{1}{8} + 25 \cdot \frac{1}{8}$$

$$= 7.5$$

ALTERNATIVELY, WE CAN DO THE FOLLOWING:

X	$p_X(u)$	$Y = g(X) = X^2$
-2	$1/8$	4
-1	$1/8$	1
0	$1/8$	0
1	$1/8$	1
2	$1/8$	4
3	$1/8$	9
4	$1/8$	16
5	$1/8$	25

$$E(Y) = 4\cdot\frac{1}{8} + 1\cdot\frac{1}{8} + 0\cdot\frac{1}{8} + 1\cdot\frac{1}{8} + 4\cdot\frac{1}{8} + 9\cdot\frac{1}{8} + 16\cdot\frac{1}{8} + 25\cdot\frac{1}{8}$$

$$= 7.5$$

$$\Rightarrow E(Y) = E(X^2) = E(g(X)) = \sum_{u_i} g(u_i)\, p_X(u_i) = \sum_{u_i} u_i^2\, p_X(u_i)$$

IN GENERAL: $E(g(X)) = \sum_{u_i} g(u_i) \cdot p_X(u_i)$: LOTUS

LAW OF UNCONSCIOUS STATISTICIAN

EXAMPLES 2.2.5 – 2.2.6

PLEASE GO THROUGH THEM YOURSELF!

PROPERTIES OF EXPECTATION

$(\Omega, \mathcal{F}, P)$ $\quad X : \Omega \to \mathbb{R}$ $\quad g(\cdot)$ and $h(\cdot)$ are functions of X.

a, b, and c are some constants.

$$E(\overbrace{a\,g(X) + b\,h(X) + c}^{\text{FUNCTION OF } X}) = \sum_{u_i} (a\,g(u_i) + b\,h(u_i) + c) \cdot p_X(u_i) \quad \because \text{LOTUS}$$

$$= \sum_{u_i} a\,g(u_i) \cdot p_X(u_i) + \sum_{u_i} b\,h(u_i) \cdot p_X(u_i) + \sum_{u_i} c \cdot p_X(u_i)$$

$$= a \sum_{u_i} g(u_i) \cdot p_X(u_i) + b \sum_{u_i} h(u_i) \cdot p_X(u_i) + c \sum_{u_i} p_X(u_i)$$

LOTUS LOTUS DEF. OF PMF

$$= a\,E(g(X)) + b\,E(h(X)) + c \cdot 1$$

$$\therefore E(a\,g(X) + b\,h(X) + c) = a\,E(g(X)) + b\,E(h(X)) + c$$

LINEARITY OF EXPECTATION

IN GENARAL : $(\Omega, \mathcal{F}, P)$ $\quad X : \Omega \to \mathbb{R}$ $\quad Y : \Omega \to \mathbb{R}$

$$E(a\,g(X,Y) + b\,h(X,Y) + c) = a\,E(g(X,Y)) + b\,E(h(X,Y)) + c$$

TAKE $g(X,Y) = X$, $h(X,Y) = Y$, $a = b = 1$, and $c = 0$

$$E(X+Y) = E(X) + E(Y)$$