ECE 313 / MATH 362 PROBABILITY WITH ENGINEERING APPLICATIONS

# LECTURE 20: RELIABILITY AND UNION BOUND

- TOPICS TO COVER (BASED ON CH 2.12)

→ RELIABILITY AND UNION BOUND

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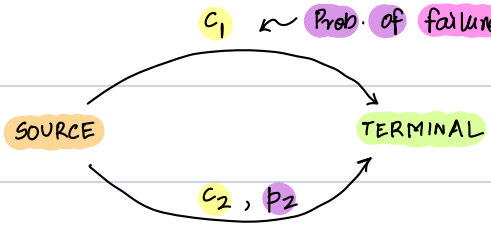
ASSUMPTION: 'ALL LINK FAILURES ARE INDEP. OF EACH OTHER'

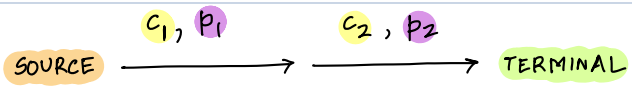
NETWORK 1:




DEF: $F_i$ : EVENT THAT LINK $c_i$ FAILS

$$P(\text{NETWORK OUTAGE}) = P(\text{BOTH } c_1 \text{ AND } c_2 \text{ FAIL}) = P(F_1 F_2) = p_1 p_2$$

NETWORK 2:



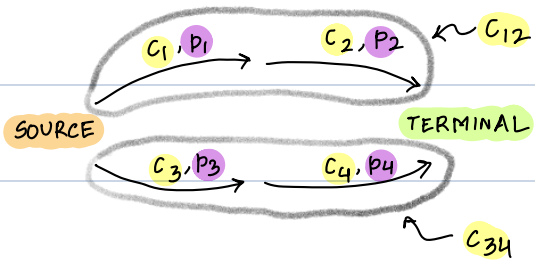

$$P(\text{NETWORK OUTAGE}) = P(\text{EITHER } c_1 \text{ OR } c_2 \text{ FAILS})$$

$$= P(F_1 \cup F_2)$$

$$= P(F_1) + P(F_2) - P(F_1 F_2)$$

$$= p_1 + p_2 - p_1 p_2$$

NETWORK 3:



$$P(\text{NETWORK OUTAGE}) = P(C_{12} \text{ AND } C_{34} \text{ FAIL})$$

$$= P(C_{12} \text{ FAILS}) \cdot P(C_{34} \text{ FAILS})$$

$$= (p_1 + p_2 - p_1 p_2)(p_3 + p_4 - p_3 p_4)$$

→ UNION BOUND :

A, B : TWO EVENTS , E.G., FAILURE EVENTS

$$P(A \cup B) \leq P(A) + P(B) \qquad \cdots (*)$$

IF A AND B ARE DISJOINT, i.e., $P(AB) = 0$

$$P(A \cup B) = P(A) + P(B)$$

IN GENERAL :

$A_1, A_2, \cdots, A_m$ : EVENTS

$$P(A_1 \cup A_2 \cup \cdots \cup A_m) \leq P(A_1) + P(A_2) + \cdots + P(A_m)$$

FOLLOWS BY INDUCTION USING (*)

WE ALSO KNOW THAT:

$$P(A \cup B) = P(A) + P(B) - P(AB)$$

IF $P(AB)$ IS SMALL THEN $P(A \cup B) \approx P(A) + P(B)$

↳ UNION BOUND IS TIGHTER!

IF $P(AB) \approx 0$ THEN $P(A \cup B) \approx P(A) + P(B)$

THIS CAN BE APPROXIMATED BY

IF A AND B ARE INDEP THEN $P(AB) \approx 0$ : $P(A) \cdot P(B) \approx 0$

$\Rightarrow$ EITHER $P(A) \approx 0$ OR $P(B) \approx 0$

IF THE PROB. OF INDIVIDUAL EVENTS ARE SMALL: UNION BOUND GIVES A GOOD APPROXIMATION

→ USING THE UNION BOUND TO APPROXIMATE NETWORK OUTAGE PROBABILITY

NETWORK 2: SOURCE —($c_1, p_1$)→ —($c_2, p_2$)→ TERMINAL

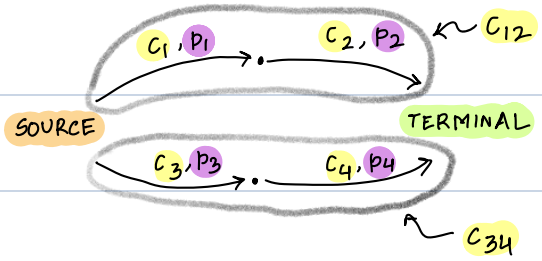
$P(\text{NETWORK OUTAGE}) = P(\text{EITHER } c_1 \text{ OR } c_2 \text{ FAILS})$

$= P(F_1 \cup F_2) = P(F_1) + P(F_2) - P(F_1 F_2)$

$= p_1 + p_2 - p_1 p_2$

UNION BOUND $\rightsquigarrow \leq P(F_1) + P(F_2) = p_1 + p_2$

NETWORK 3:




EXACT CALCUATION

$$P(\text{NETWORK OUTAGE}) = P(c_{12} \text{ AND } c_{34} \text{ FAIL})$$

$$= P(c_{12} \text{ FAILS}) \cdot P(c_{34} \text{ FAILS})$$

$$= (p_1 + p_2 - p_1 p_2)(p_3 + p_4 - p_3 p_4)$$

ALSO:

$$\text{NETWORK OUTAGE} = c_{12} \text{ AND } c_{34} \text{ FAIL}$$

$$= c_{12} \text{ FAILS AND } c_{34} \text{ FAILS}$$

$$= (c_1 \text{ OR } c_2 \text{ FAILS}) \text{ AND } (c_3 \text{ OR } c_4 \text{ FAILS})$$

$$= (F_1 \cup F_2) \cap (F_3 \cup F_4)$$

$$= (F_1 F_3 \cup F_1 F_4 \cup F_2 F_3 \cup F_2 F_4)$$

$$\Rightarrow P(\text{NETWORK OUTAGE}) = P(F_1 F_3 \cup F_1 F_4 \cup F_2 F_3 \cup F_2 F_4)$$

UNION BOUND APPROXIMATION

$$\leq P(F_1 F_3) + P(F_1 F_4) + P(F_2 F_3) + P(F_2 F_4)$$

$$= p_1 p_3 + p_1 p_4 + p_2 p_3 + p_2 p_4$$

$$= (p_1 + p_2)(p_3 + p_4)$$