ECE 313 / MATH 362 PROBABILITY WITH ENGINEERING APPLICATIONS

LECTURE 2 : AXIOMS OF PROBABILITY

- TOPICS TO COVER (BASED ON CH 1.2)
  - → AXIOMS OF PROBABILITY

→ AXIOMS OF PROBABILITY

- RANDOM EXPERIMENT (RE) : ROLLING A DIE
- SAMPLE SPACE (Ω) : $\{1, 2, 3, 4, 5, 6\}$: COLLECTION OF ALL POSSIBLE OUTCOMES
- EVENT (E) : A SUBSET OF THE SAMPLE SPACE
  - ELEMENTARY EVENT (e) : $\{1\}$, $\{2\}$, $\{3\}$, ..
- PROBABILITIES : $P(\{1\}) = P(\{2\}) = \dots = P(\{6\}) = \frac{1}{6} := P\{i\} \quad i = 1, 2, \dots, 6$
- UNION AND INTERSECTION OF EVENTS: (SYMBOL : $\cap$, , $\cdot$)

SYMBOL : $\cup$

- $E_1 \cup E_2$: EVENT

E.G. $E_1 = \{2\}$ $E_2 = \{4, 6\}$

$E_1 \cup E_2 = \{2, 4, 6\}$: EVEN NUMBER $P(E_1 \cup E_2) = 1/2$

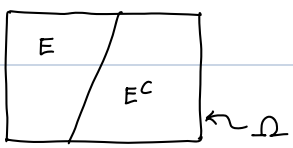
- $E_1 \cap E_2$: EVENT

E.G. $E_1 = \{2\}$ $E_2 = \{4, 6\}$

$E_1 \cap E_2 = \phi$ $P(E_1 \cap E_2) = P(\phi) = 0$

- COMPLEMENT OF AN EVENT : EVENT

SYMBOL : $-$, ^{c} $E = \{1, 2, 3\}$ $E^c = \{4, 5, 6\}$



- **MUTUALLY EXCLUSIVE** EVENTS : $E_1 \cap E_2 = \phi$

  $E_1, E_2, \ldots, E_n$ are **MUTUALLY EXCLUSIVE** IF $E_i \cap E_j = \phi \quad \forall \quad i \neq j$

- **EXHAUSTIVE** EVENTS :

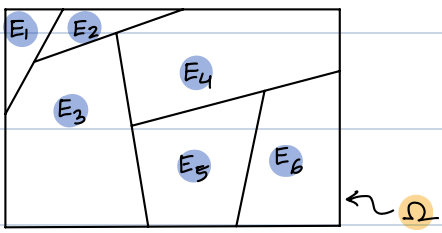
  $E_1, E_2, \ldots, E_n$ are **EXHAUSTIVE** IF $E_1 \cup \ldots \cup E_n = \Omega$

$E_1 = \{1, 2, 3\}$

$E_2 = \{4, 5, 6\}$

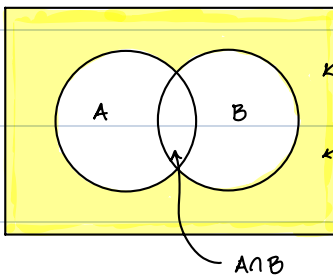
} **MUTUALLY EXCLUSIVE AND EXHAUSTIVE**

'**PARTITION OF THE SAMPLE SPACE**'



- THE COLLECTION OF **ELEMENTARY EVENTS** ALWAYS FORMS A **PARTITION**
- THE COLLECTION **E, E^c** ALWAYS MAKES A **PARTITION**

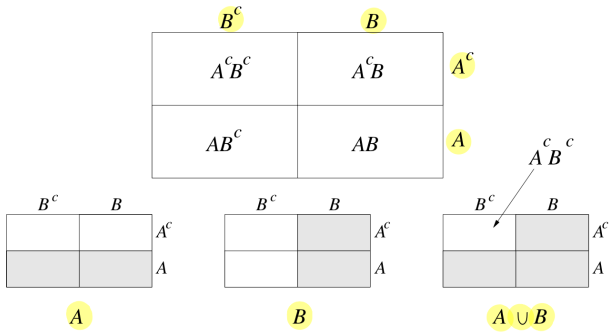
- **DE MORGAN'S LAW** :
  - $(A \cup B)^c = A^c \cap B^c \Rightarrow A \cup B = (A^c \cap B^c)^c$
  - $(A \cap B)^c = A^c \cup B^c$



$(A \cup B)^c = A^c \cap B^c$ (OBSERVE!)

DRAW THE SAME FOR THE SECOND VERSION OF THE DE MORGAN'S RULE

'**VENN DIAGRAM**'



NOTE THAT $A \cup B$ ONLY DOES NOT INCLUDE $A^C B^C$, i.e., $(A \cup B)^C = A^C B^C$

Figure 1.1: Karnaugh map showing how two sets, A and B, partition Ω.

- ALGEBRAIC PROOF :

$$(A \cup B)^C = \{\omega \in \Omega : \omega \notin A \cup B\}$$

$$= \{\omega \in \Omega : \omega \notin A \text{ or } \omega \notin B\}$$

$$= \{\omega \in \Omega : \omega \in A^C \text{ and } \omega \in B^C\}$$

$$= \{\omega \in \Omega : \omega \in A^C \cap B^C\}$$

$$= A^C \cap B^C$$

- AXIOMS OF PROBABILITY :

- PROBABILITY SPACE : $(\Omega, \mathcal{F}, P)$

SCRIPT F : A SIGMA FIELD OVER Ω

PROBABILITY MEASURE OVER SETS IN $\mathcal{F}$

SAMPLE SPACE

- EVENT AXIOMS :

E.1 Ω IS AN EVENT, i.e., $\Omega \in \mathcal{F}$.

E.2 IF A IS AN EVENT, SO IS A^C, i.e., IF $A \in \mathcal{F}$, THEN $A^C \in \mathcal{F}$.

E.3 IF A and B ARE EVENTS, SO IS $A \cup B$, i.e., IF $A, B \in \mathcal{F} \Rightarrow A \cup B \in \mathcal{F}$.