ECE 313 / MATH 362 PROBABILITY WITH ENGINEERING APPLICATIONS

LECTURE 18: BINARY HYPOTHESIS TESTING

- TOPICS TO COVER (BASED ON CH 2.11)
  - → INTRODUCTION TO BINARY HYPOTHESIS TESTING FOR DISCRETE-TYPE OBSERVATIONS
  - → MAXIMUM LIKELIHOOD (ML) DECISION RULE

→ INTRODUCTION TO BINARY HYPOTHESIS TESTING FOR DISCRETE-TYPE OBSERVATIONS

PROBABILISTIC / STATISTICAL INFERENCE

Q: WHAT CAN YOU SAY ABOUT THE UNDERLYING PROCESS / DISTRIBUTION BASED ON SOME RANDOM OBSERVATIONS?

PARAMETER ESTIMATION

Q: GIVEN SOME RANDOM OBSERVATIONS, CAN YOU ESTIMATE PARAMETERS OF THE UNDELYING DISTRIBUTION?

POINT ESTIMATION

Q: WHAT'S YOUR ESTIMATE AS A SINGLE VALUE?

E.G. MAXIMUM LIKELIHOOD ESTIMATION (MLE)

INTERVAL ESTIMATION

Q: WHAT'S YOUR ESTIMATE AS A RANGE / INTERVAL?

E.G. CONFIDENCE INTERVAL

HYPOTHESIS TESTING

Q: GIVEN SOME RANDOM OBSERVATIONS, CAN YOU ESTIMATE WHAT'S THE STATE OF PARAMETERS/ DISTRIBUTION FROM SOME COMPETING CHOICES?

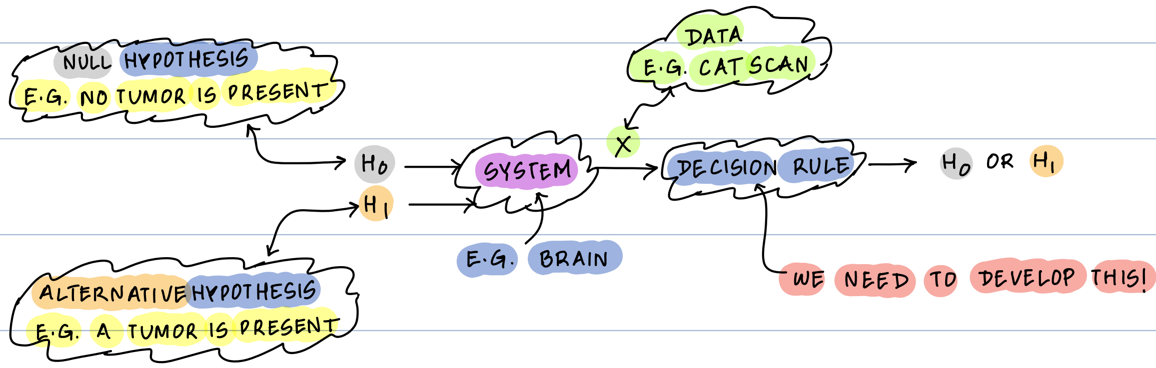
IF TWO CHOICES:

BINARY HYPOTHESIS TESTING

E.G.:

MAXIMUM LIKELIHOOD (ML) DECISION RULE

MAXIMUM A POSTERIORI (MAP) DECISION RULE



H_0 : NULL HYPOTHESIS : IF H_0 IS TRUE, THE PMF OF X IS p_0

H_1 : ALTERNATIVE HYPOTHESIS : IF H_1 IS TRUE, THE PMF OF X IS p_1

LIKELIHOOD MATRIX :

E.G. $P(\{X = x \mid H_i\})$, $i = 0, 1$

	$X=0$	$X=1$	$X=2$	$X=3$	
H_1	0.0	0.1	0.3	0.6	$\rightsquigarrow p_1$
H_0	0.4	0.3	0.2	0.1	$\rightsquigarrow p_0$

DECISION RULE : $D(X)$: DECISION RULE / TEST IS A FUNCTION OF X ← DATA

E.G. DECISION RULE : DECLARE H_1 IF $X \geq 1$

OBSERVATION : $X = 2$

DECISION : H_1

	$X=0$	$X=1$	$X=2$	$X=3$
H_1	0.0	$\underline{0.1}$	$\underline{0.3}$	$\underline{0.6}$
H_0	$\underline{0.4}$	0.3	0.2	0.1

DECISION (BASED ON X)

		$\hat{H}_0$	$\hat{H}_1$
TRUE STATE	H_0	☺	☹
	H_1	☹	☺

P(TYPE I ERROR : FALSE ALARM) $= p_{false\ alarm}$ = LEVEL OF SIGNIFICANCE (α)

$p_{miss} = P$(TYPE II ERROR : MISS) : $1 -$ POWER OF THE TEST $= \beta$

TYPE I ERROR : FALSE ALARM = H_1 IS TRUE | H_0

REJECT H_0 | H_0

P(FALSE ALARM) $:= p_{false\ alarm} = P(H_1$ IS TRUE $| H_0)$

TYPE II ERROR : MISS = H_0 IS TRUE | H_1

ACCEPT H_0 | H_1

P(MISS) $:= p_{miss} = P(H_0$ IS TRUE $| H_1)$

DECISION : H_0 — DECISION : H_1

	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	0.0	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

p_{miss}

$0.3 + 0.2 + 0.1 = 0.6 = p_{false\ alarm}$

A GOOD DECISION RULE / TEST SHOULD MINIMIZE $p_{false\ alarm}$ AND p_{miss} !

NOTE THAT YOU CANNOT MINIMIZE BOTH AT THE SAME TIME.

→ MAXIMUM LIKELIHOOD (ML) DECISION RULE

E.G. $P(X=x \mid H_i)$; $i=0,1$

		$X=0$	$X=1$	$X=2$	$X=3$
$p_1(x) : P(X=x \mid H_1)$	⇝ H_1	0.0	0.1	<u>0.3</u>	<u>0.6</u>
$p_0(x) : P(X=x \mid H_0)$	⇝ H_0	<u>0.4</u>	<u>0.3</u>	0.2	0.1

DECISION RULE : GIVEN $X=x$, COMPARE $p_1(x)$ AND $p_0(x)$

IF $p_1(x) > p_0(x)$: DECLARE H_1

OTHERWISE : H_0

ANOTHER WAY : IF $\dfrac{p_1(x)}{p_0(x)} > 1$: DECLARE H_1

OTHERWISE : H_0

CONDITIONAL LIKELIHOODS

DEFINE $\Lambda(X=x) = \dfrac{p_1(x)}{p_0(x)}$ ⇜ LIKELIHOOD RATIO

IF $\Lambda(X=x) > 1$: DECLARE H_1

OTHERWISE : H_0

IN GENERAL :

τ IS A USER-DEFINED THRESHOLD

IF $\Lambda(X=x) > \tau$: DECLARE H_1

OTHERWISE : H_0