ECE 313 / MATH 362 PROBABILITY WITH ENGINEERING APPLICATIONS

LECTURE 17 : CONFIDENCE INTERVAL

- TOPICS TO COVER (BASED ON CH 2.9)
  - → CONFIDENCE INTERVAL

→ CONFIDENCE INTERVAL

LET $X \sim$ BINOMIAL (n, p) : $\mu = np$ $\sigma^2 = np(1-p)$

RECALL THE CHEBYCHEV INEQUALITY :

$$P\{ |X - \mu| \geq a\sigma \} \leq \frac{1}{a^2}$$

$$P\{ |X - np| \geq a\sqrt{np(1-p)} \} \leq \frac{1}{a^2}$$

$$\Rightarrow P\left\{ \left|\frac{X}{n} - p\right| \geq a\frac{\sqrt{np(1-p)}}{n} \right\} \leq \frac{1}{a^2}$$

$$P\left\{ \left|\frac{X}{n} - p\right| \geq a\sqrt{\frac{p(1-p)}{n}} \right\} \leq \frac{1}{a^2}$$

$$\Rightarrow P\left\{ \left|\frac{X}{n} - p\right| < a\sqrt{\frac{p(1-p)}{n}} \right\} \geq 1 - \frac{1}{a^2}$$

HOW? EXERCISE!

$\hat{p}_{MLE}$

$$P\{ p \in \underbrace{\left(\frac{X}{n} - a\sqrt{\frac{p(1-p)}{n}}, \frac{X}{n} + a\sqrt{\frac{p(1-p)}{n}} \right)}_{\text{RANDOM AS } X \text{ IS RANDOM}} \} \geq 1 - \frac{1}{a^2}$$

PARAMETER OF THE BINOMIAL DIST.

OBSERVE THAT $\max_p p(1-p) = \frac{1}{2}(1-\frac{1}{2}) = \frac{1}{4}$ $p \in [0,1]$

$\hat{p}_{MLE}$

$$P\left\{ p \in \underbrace{\left(\frac{X}{n} - a\sqrt{\frac{1/4}{n}} \,,\, \frac{X}{n} + a\sqrt{\frac{1/4}{n}} \right)}_{\text{RANDOM AS } X \text{ IS RANDOM}} \right\} \geq 1 - \frac{1}{a^2}$$

PARAMETER OF THE BINOMIAL DIST.

TAKE $a = 5$:

$\hat{p}_{MLE}$

$$P\left\{ p \in \underbrace{\left(\frac{X}{n} - 5\sqrt{\frac{1/4}{n}} \,,\, \frac{X}{n} + 5\sqrt{\frac{1/4}{n}} \right)}_{\text{RANDOM AS } X \text{ IS RANDOM}} \right\} \geq 1 - \frac{1}{5^2} = .96$$

PARAMETER OF THE BINOMIAL DIST.

: 96% CONFIDENCE INTERVAL

E.G. YOU TOSS A BIASED COIN 10 TIMES AND OBSERVE 6 HEADS.

X = # OF HEADS IN $n = 10$ TRIALS

$X \sim \text{BINOMIAL}(\overset{10}{n}, p)$

$$\hat{p}_{MLE} = \frac{x}{n} = \frac{6}{10}$$

96% CONFIDENCE INTERVAL FOR p : $\left(\hat{p}_{MLE} - \frac{5}{2\sqrt{n}} \,,\, \hat{p}_{MLE} + \frac{5}{2\sqrt{n}} \right)$

$$\frac{5}{2\sqrt{n}} = \frac{5}{2\sqrt{10}}$$

$\Rightarrow$ 96% CONFIDENCE INTERVAL FOR p : $\left(\frac{6}{10} - \frac{5}{2\sqrt{10}} \,,\, \frac{6}{10} + \frac{5}{2\sqrt{10}} \right)$