ECE 313 / MATH 362 PROBABILITY WITH ENGINEERING APPLICATIONS

LECTURE 16 : MARKOV AND CHEBYCHEV INEQUALITIES

- TOPICS TO COVER (BASED ON CH 2.9)

  → MARKOV INEQUALITY

  → CHEBYCHEV INEQUALITY

→ MARKOV INEQUALITY

IF : Y IS A NON-NEGATIVE RV, i.e., $Y \geq 0$

$c > 0$ IS SOME CONSTANT

THEN :
$$P\{Y \geq c\} \leq \frac{E(Y)}{c}$$

PROOF:
$$E(Y) := \sum_i u_i \, p_Y(u_i)$$

$$= \sum_{(i:\, u_i < c)} u_i \, p_Y(u_i) + \sum_{(i:\, u_i \geq c)} u_i \, p_Y(u_i)$$

$(i: u_i < c \; : \; u_i > 0)$ $\qquad$ $(i: u_i \geq c \; : \; u_i \geq c)$

$$\geq \sum_{i:\, u_i < c} 0 \, p_Y(u_i) + \sum_{i:\, u_i \geq c} c \, p_Y(u_i)$$

$$= 0 + c \sum_{i:\, u_i \geq c} p_Y(u_i)$$

$$\Rightarrow E(Y) \leq c \cdot P\{Y \geq c\}$$

$$\Rightarrow P\{Y \geq c\} \leq \frac{E(Y)}{c}$$ HENCE THE INEQUALITY.

→ CHEBYCHEV INEQUALITY

LET X BE A RV WITH FINITE MEAN μ AND VARIANCE σ^2, THEN FOR ANY $d > 0$:

$$P\{|X-\mu| \geq d\} \leq \frac{\sigma^2}{d^2}$$

PROOF: RECALL MARKOV INEQUALITY

$$P\{Y \geq c\} \leq \frac{E(Y)}{c} \qquad Y \geq 0 \text{ AND } c > 0$$

TAKE $Y = |X-\mu|^2 \geq 0$

$c = d^2 > 0$

$$\Rightarrow P\{\underbrace{|X-\mu|^2 \geq d^2}_{\text{EQUIVALENT}}\} \leq \frac{E(X-\mu)^2}{d^2} = \frac{\sigma^2}{d^2}$$

$$\Rightarrow P\{|X-\mu| \geq d\} \leq \frac{\sigma^2}{d^2}$$

TAKE $d = a\sigma > 0$

$$\Rightarrow P\{|X-\mu| \geq a\sigma\} \leq \frac{1}{a^2}$$

HENCE THE INEQUALITY.

ANOTHER FORM OF CHEBYCHEV INEQUALITY:

$$P\{|X-\mu| < a\sigma\} \geq 1 - \frac{1}{a^2}$$