LECTURE 15 : MAXIMUM LIKELIHOOD ESTIMATION (MLE)

- TOPICS TO COVER (BASED ON CH 2.8)

  → MAXIMUM LIKELIHOOD ESTIMATION (MLE)

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$X \sim p_\theta$

RV — PMF — parameter

E.G. $X_1 \sim \text{Binomial}(n, p)$ — parameters $\qquad p_{n,p}(k) = \binom{n}{k} p^k (1-p)^{n-k}$

$X_2 \sim \text{Poisson}(\lambda)$ — parameter $\qquad p_\lambda(k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$

p_θ : DISTRIBUTION OR PMF WITH UNKNOWN θ $\qquad$ E.G. p_θ : BERNOULLI(p)

→ X : AN OBSERVATION FROM p_θ $\qquad$ → X : A SUCCESS

GIVEN X WHAT'S YOUR 'BEST' GUESS ABOUT θ ?

GUESS — ESTIMATE

WE OBSERVE $X = k$ FROM p_θ : THE PMF

KNOWN — UNKNOWN

THE 'LIKELIHOOD' THAT $X = k$ IS $p_\theta(k)$.

LIKELIHOOD — PROBABILITY

WE GUESS/ESTIMATE θ SUCH THAT THE LIKELIHOOD OF $X=k$ IS MAXIMUM AMONG ALL POSSIBILITIES OF θ.

$$\hat{\theta}_{MLE} = \arg\max_{\theta} p_{\theta}(k)$$

- MLE FOR BERNOULLI DIST.

$$X \sim \text{BERNOULLI}(p)$$

$$p_X(x) = p^x (1-p)^{1-x} \quad : \text{PMF}$$

$$L(p) = p^x (1-p)^{1-x} \quad : \text{LIKELIHOOD FUNCTION}$$

WE WANT TO SOLVE : $\hat{p}_{MLE} = \arg\max_{p} p^x (1-p)^{1-x}$

$$L(p) = p^x (1-p)^{1-x}$$

$$\Rightarrow \ln L(p) = x \ln p + (1-x) \ln (1-p)$$

$$\frac{\partial}{\partial p} \ln L(p) = \frac{x}{p} + \frac{-(1-x)}{(1-p)} = 0$$

$$\Rightarrow \hat{p}_{MLE} = x$$

- MLE FOR BINOMIAL DIST.

ASSUME : n IS KNOWN

$$X \sim \text{BINOMIAL}(n, p)$$

$$p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$L(p) = \binom{n}{x} p^x (1-p)^{n-x}$$

WE WANT TO SOLVE : $\hat{p}_{MLE} = \underset{p}{\arg\max} \binom{n}{x} p^x (1-p)^{n-x}$

$$L(p) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$\Rightarrow \ln L(p) = \ln \binom{n}{x} + x \ln p + (n-x) \ln(1-p)$$

$$\frac{\partial}{\partial p} \ln L(p) = 0 + \frac{x}{p} + - \frac{(n-x)}{(1-p)} = 0$$

$$\Rightarrow \hat{p}_{MLE} = \frac{x}{n}$$

- MLE FOR GEOMETRIC DIST.

$$X \sim \text{GEOMETRIC}(p)$$

$$p_X(x) = (1-p)^{x-1} p$$

$$L(p) = (1-p)^{x-1} p$$

WE WANT TO SOLVE : $\hat{p}_{MLE} = \underset{p}{\arg\max} (1-p)^{x-1} p$

$$L(p) = (1-p)^{x-1} p$$

$$\Rightarrow \ln L(p) = (x-1) \ln(1-p) + \ln p$$

$$\frac{\partial}{\partial p} \ln L(p) = - \frac{(x-1)}{(1-p)} + \frac{1}{p} = 0$$

$$\Rightarrow \hat{p}_{MLE} = \frac{1}{x}$$

- MLE FOR POISSON DIST.

$$X \sim \text{POISSON}(\lambda)$$

$$p_X(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$L(\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

WE WANT TO SOLVE : $\hat{\lambda}_{MLE} = \underset{\lambda}{\arg\max} \; \frac{e^{-\lambda}\lambda^{x}}{x!}$

$$L(\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$\Rightarrow \quad \ln L(\lambda) = -\lambda + x \ln \lambda - \ln x!$$

$$\frac{\partial}{\partial \lambda} \ln L(\lambda) = -1 + \frac{x}{\lambda} = 0$$

$$\Rightarrow \quad \hat{\lambda}_{MLE} = x$$

- MLE FOR NEG. BINOMIAL DIST.

  → EXERCISE !