ECE 313: Problem Set 11

Due: Friday, November 21 at 7:00:00 p.m.
Reading: *ECE 313 Course Notes,* Sections 4.4 – 4.6
Note on reading: For most sections of the course notes, there are short-answer questions at the end of the chapter. We recommend that after reading each section, you try answering the short-answer questions. Do not hand in; answers to the short answer questions are provided in the appendix of the notes.
Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted. Please write at the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned pages.**

1. [**Joint PDF and Independence**]
   Let (X, Y) be continuous random variables with joint probability density function (PDF) given by:
   $$f_{X,Y}(x,y) = \begin{cases} 6(1-y), & 0 \leq x \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

   Answer the following questions about X and Y.

   (a) Verify that $f_{X,Y}(x,y)$ is a valid joint PDF.
   (b) Find the marginal PDFs $f_X(x)$ and $f_Y(y)$.
   (c) Find the conditional PDF $f_{X|Y}(x|y)$.
   (d) Find $\mathbb{E}[X|Y = y]$.
   (e) Are X and Y independent? Justify your answer.

2. [**Joint PDF and Independence**]
   Let (X, Y) be continuous random variables with joint probability density function (PDF) given by:
   $$f_{X,Y}(x,y) = \begin{cases} \frac{xy}{c}, & \text{if } x \in [0,1] \text{ and } y \in [0,2] \\ \frac{3y-xy}{c} & \text{if } x \in [2,3] \text{ and } y \in [0,2] \\ 0, & \text{otherwise} \end{cases}$$

   Answer the following questions about X and Y.

   (a) Compute c.
   (b) Find the marginal PDFs $f_X(x)$ and $f_Y(y)$.

(c) Find the conditional PDF $f_{X|Y}(x)$ and decide if X and Y are independent.

3. [**Sum of Two Independent Continuous-type Random Variables**]
   Suppose X and Y have the joint pdf

   $$f_{X,Y}(u,v) = \begin{cases} 4uv, & \text{if } 0 \le u \le 1,\ 0 \le v \le 1; \\ 0, & \text{otherwise.} \end{cases}$$

   Let $S = X + Y$. Find the pdf of S, i.e., find $f_S(s)$.

   (a) Find the pdf of S, i.e., find $f_S(s)$. *Hint:* You may want to consider the cases $s < 0$, $0 \le s < 1$, $1 \le s \le 2$, and $s > 2$ separately.

   (b) Confirm that $f_S(s)$ integrates to 1 and therefore is a valid pdf.

4. [**Sum of Two Continuous-type Random Variables**]
   Suppose X and Y have the joint pdf

   $$f_{X,Y}(u,v) = \begin{cases} 2v, & 0 \le u \le 1,\ 0 \le v \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

   Let $S = X + Y$.

   (a) Are X and Y independent? Justify your answer.

   (b) Find the pdf of S, i.e., compute $f_S(s)$.