**University of Illinois** **Fall 2025**

# ECE 313: Problem Set 2

**Due:** Friday, September 19 at 07:00:00 p.m.
**Reading:** *ECE 313 Course Notes,* Sections 2.1 - 2.4.2.

**Note on reading:** For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.

**Note on turning in homework:** Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.
Please write on the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned pages.**

1. **[PMF, Mean, and Variance]**
   You roll a fair coin and a fair die simultaneously. Let $X = 1$ ($X = 0$) if the coin shows a head (tail) and Y equals the number showing on the die. Furthermore, let $Z = 2X + Y$.

   (a) Find the pmfs of $X$, $Y$, and $Z$.

   (b) Find $E[X]$, $E[Y]$, and $E[Z]$ using the pmfs of $X$, $Y$, and $Z$, respectively. Can you guess a general relationship between $E[X]$, $E[Y]$, and $E[Z]$?

   (c) Find $\mathrm{Var}(X)$, $\mathrm{Var}(Y)$, and $\mathrm{Var}(Z)$. Can you guess a general relationship between $\mathrm{Var}(X)$, $\mathrm{Var}(Y)$, and $\mathrm{Var}(Z)$?

   (d) Find $E[1/Z]$ and $\mathrm{Var}(1/Z)$?

2. **[PMF, Mean and standard deviation]**
   Suppose two fair dice are rolled independently, so the sample space is $\Omega = \{(i, j) : 1 \leq i \leq 6, \text{ and } 1 \leq j \leq 6\}$, and all outcomes are equally likely. Let $X$ be the random variable defined by $X(i, j) = i - j$, and let $Y$ be the random variable defined by $Y(i, j) = \max\{0, i - j\}$ (Rectified Linear Unit or ReLU).

   (a) Derive the pmf of $X$ and sketch it.

   (b) Find the mean, $E[X]$, and standard deviation, $\sigma_X$, of $X$.

   (c) Derive the pmf of $Y$ and sketch it.

   (d) Find the mean, $E[Y]$, and standard deviation, $\sigma_Y$, of $Y$.

3. **[PMF, Mean]**
   Suppose that you as a UIUC student are planning on going to a concert in the Krannert Center. Ten of your friends are also interested in the same concert. Four of your friends are

UIUC students, whereas the remaining six friends are not students. You randomly choose three of your friends to go with you. When you get to the Krannert Center, it turns out that UIUC students pay $10 and those who are not students pay $50 for their tickets. Let $X$ be the total amount of money you and your friends end up paying to get into the concert.

(a) What values can $X$ take?

(b) Find the pmf of $X$.

(c) Find the expected value of $X$.

4. **[Matching cards to boxes]**
Three boxes are placed on a table, with the $i$-th box containing a card with the number $i$, for $i = 1, 2, 3$. The cards are then removed from the boxes, combined with card with number 4 on it, randomly shuffled, and three of the cards are randomly selected and placed one each into the three boxes; all possibilities of which card is placed in which box are equally likely. Let $X$ denote the number of boxes that get back their original card.

(a) Describe a suitable sample space $\Omega$ to describe the experiment. How many elements does $\Omega$ have?

(b) Find the pmf of $X$.

(c) Find $E[X]$.

(d) Find $\mathrm{Var}(X)$.

5. **[Illini T-Shirts]**
A bag contains 4 orange and 6 blue t-shirts. Two t-shirts are chosen from the bag at random. Let $X$ denote the number of blue t-shirts chosen.

(a) Find the pmf of $X$.

(b) Find $E[(X + 1)(X + 2)]$.

6. **[Expected Value]**
If $E[(X + 2)^2] = 4$ and $\mathrm{Var}(X) = 1$, find $E[X^2]$ and $E[X]$.