University of Illinois **Fall 2025**

ECE 313: Problem Set 1

Due: Friday, September 12 at 07:00:00 p.m.
Reading: *ECE 313 Course Notes,* Chapter 1

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.

Please write on the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET #

Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned pages.**

1. **[Defining a set of outcomes I]**
   Ten balls, numbered one through ten, are initially in a bag. Three balls are drawn out, one at a time, without replacement.

   (a) Define a sample space Ω describing the possible outcomes of this experiment. To be definite, suppose the order the three balls are drawn out is *unimportant*. Explain how the elements of your set correspond to outcomes of the experiment.

   (b) What is the cardinality of Ω?

2. **[Defining a set of outcomes II]**
   A random experiment consists of selecting two balls in succession from an urn containing two blue balls and one red ball. Assume that each ball from the urn is equally likely to be chosen.

   (a) Suppose that the balls are not replaceable, i.e., the chosen ball in the first selection is removed from the urn. What is the sample space Ω for this experiment?

   (b) Suppose now that the balls are replaceable, i.e., the chosen ball in the first selection is immediately put back into the urn. What is the sample space Ω for this experiment?

   (c) Considering both of these experiments, does the outcome of the first draw affect the outcome of the second draw? Please briefly justify your answer for both cases.

3. **[Using set theory to calculate probabilities of events]**
   Suppose A and B are two events defined on a probability space with $P(A) = 5/6$ and $P(B) = 1/2$.

   (a) If $B \subset A$, calculate $P(AB)$.

   (b) If $A \cup B = \Omega$, calculate $P(AB)$.

4. **[Possible probability assignments]**
A random experiment has a sample space $\Omega = \{a, b, c, d\}$. Suppose that $P(\{b, c, d\}) = \frac{5}{6}$, $P(\{a, b\}) = \frac{1}{3}$, and $P(\{b, c\}) = \frac{1}{2}$. Use the axioms of probability to find the probabilities of the elementary events ($P(\{a\})$, $P(\{b\})$, $P(\{c\})$, and $P(\{d\})$).

5. **[Displaying outcomes in a two event Karnaugh map]**
Two fair dice are rolled. Let A be the event the sum is 4 and B be the event at least one of the numbers rolled is strictly less than 3.

   (a) Display the outcomes in a Karnaugh map.

   (b) Determine $P(AB)$.

6. **[Principles of Counting]**
A restaurant offers 5 entrees: $\{E_1, E_2, E_3, E_4, E_5\}$ and 8 sides: $\{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8\}$. Alice and Bob want to choose 3 different entrees and 4 different sides as their dinner.

   (a) How many different dinners consisting of 3 different entrees and 4 different sides are possible?

   (b) Suppose that side from $\{S_1, S_2, S_3, S_4\}$ can only be ordered if E_1 is chosen as one of their entrees, and side from $\{S_5, S_6, S_7, S_8\}$ can be only ordered if both E_2 and E_3 are ordered as two of their entrees. With these constraints, how many different dinners consisting of 3 different entrees and 4 different sides are possible?

7. **[Two more poker hands]**
Suppose five cards are drawn from a standard 52 card deck of playing cards, as described in Example 1.4.3, with all possibilities being equally likely.

   (a) *FLUSH* is the event that all five cards have the same suit. Find $P(FLUSH)$.

   (b) *SPECIAL* is the event that the five cards are one of A, K, Q, J but not necessarily of the same suit. Find $P(SPECIAL)$.