ECE 313: Lecture 8
Independent events and independent random variables
Mean, variance, LOTUS (revisit)

i_L , i_R

$i \in \{1, 2, \ldots, n\}$

Example: n pairs of shoes, each $\in \{L, R\}$

Pick 2 shoes at random

1. What is the prob getting a matching pair (i_L, i_R) for same i

2. " " " getting a L & R $(i_L, j_R$ for $i, j)$

Solution 1. Total # of possible sets of 2 shoes

$$|\Omega| = \binom{2n}{2} = \frac{2n(2n-1)}{2}$$

$$M = \{(1_L, 1_R), (2_L, 2_R), \ldots (n_L, n_R)\}$$

$$P(M) = \frac{|M|}{|\Omega|} = \frac{n \cdot 2}{2n(2n-1)} = \frac{1}{2n-1}$$

Alternatively: E_1 = "pick the first shoe"

E_2 = "pick the second shoe that match w. the first one"

We need:

$$P(E_1 E_2) = P(E_1)\, P(E_2 \mid E_1)$$

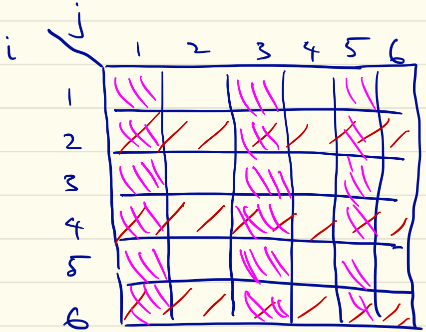
$$= 1 \cdot \frac{1}{2n-1} = \frac{1}{2n-1}$$

(2) $$P(\{\text{one } L, \text{ one } R\}) = \frac{n^2}{\binom{2n}{2}} = \frac{2n^2}{2n(2n-1)}$$

$$= \frac{n}{2n-1}$$

Alternatively $= \dfrac{n}{2n-1}$

Ex 2 : Throw 2 dices $\qquad \Omega = \{(i,j) : 1 \le i,j \le 6\}$



A = " First dice is even" $\qquad A =$

B = " Sum of 2 dices is odd"

C = " Second dice is odd" $\qquad C =$

$$P(A) = \frac{18}{36} = \frac{1}{2}$$

$$P(C) = \frac{18}{36} = \frac{1}{2}$$

$$P(\underbrace{A \cap C}_{AC}) = \frac{9}{36} = \frac{1}{4}$$

A, C

In this case

$$P(AC) = P(A)P(C)$$

$$\Rightarrow \underbrace{\frac{P(AC)}{P(C)}}_{P(A|C)} = P(A)$$

Definition: Two events A & B are mutually independent if

$$P(AB) = P(A)\,P(B)$$

Exercise: Is A & B independent in the previous example

X is a discrete random variable

$$P(\{X = k\}) = f_X(k)$$

$$E[X] = \sum_k k \underbrace{f_X(k)}_{P(\{X = k\})}$$

$$E[\underbrace{X^2}_{Y}] = \sum_k \underbrace{k^2}_{y = k^2} f_X(k)$$

$$\sigma_X^2 = Var[X] = E[X^2] - (E[X])^2$$

(LOTUS)

$$E[g(X)] = \sum_k g(k) f_X(k)$$