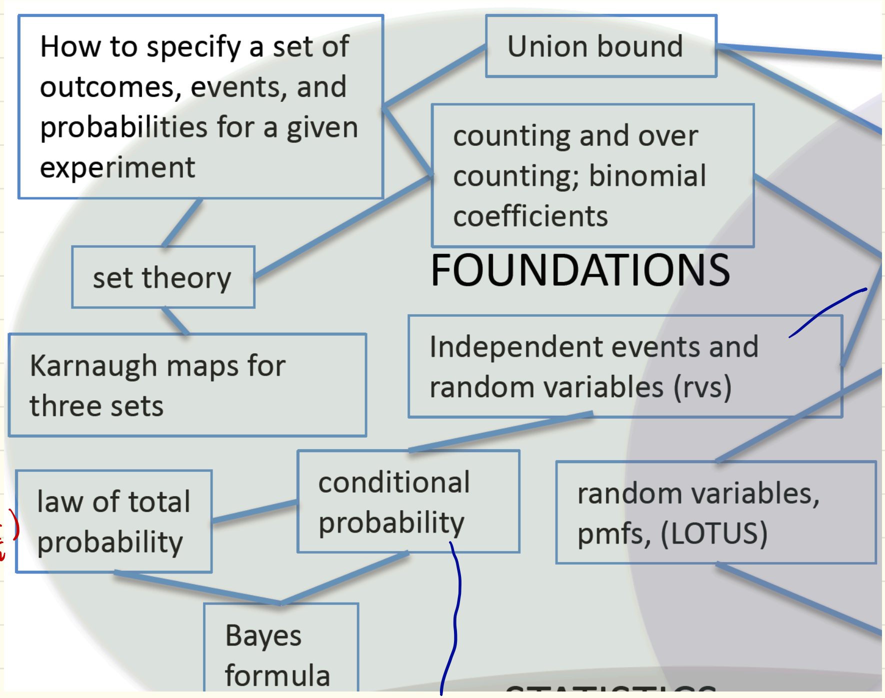
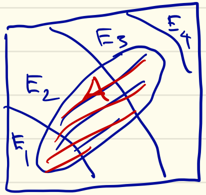
ECE 313: Lecture 42: Review and Wrap-up





$$P(A) = \sum_i \underbrace{P(AE_i)}_{P(E_i)P(A|E_i)}$$

where

$$\cup E_i = \Omega$$

$$E_i \cap E_j = \phi$$

Independent

$$P(AB) = P(A)P(B)$$

$$f_{X,Y}(u,v) = f_X(u) f_Y(v)$$

$$P(AB) = P(A)\,P(B|A)$$

$$f_{X,Y}(u,v) = f_X(u)\, f_{Y|X}(v|u)$$

$B_i \sim \mathrm{Bern}(p)$

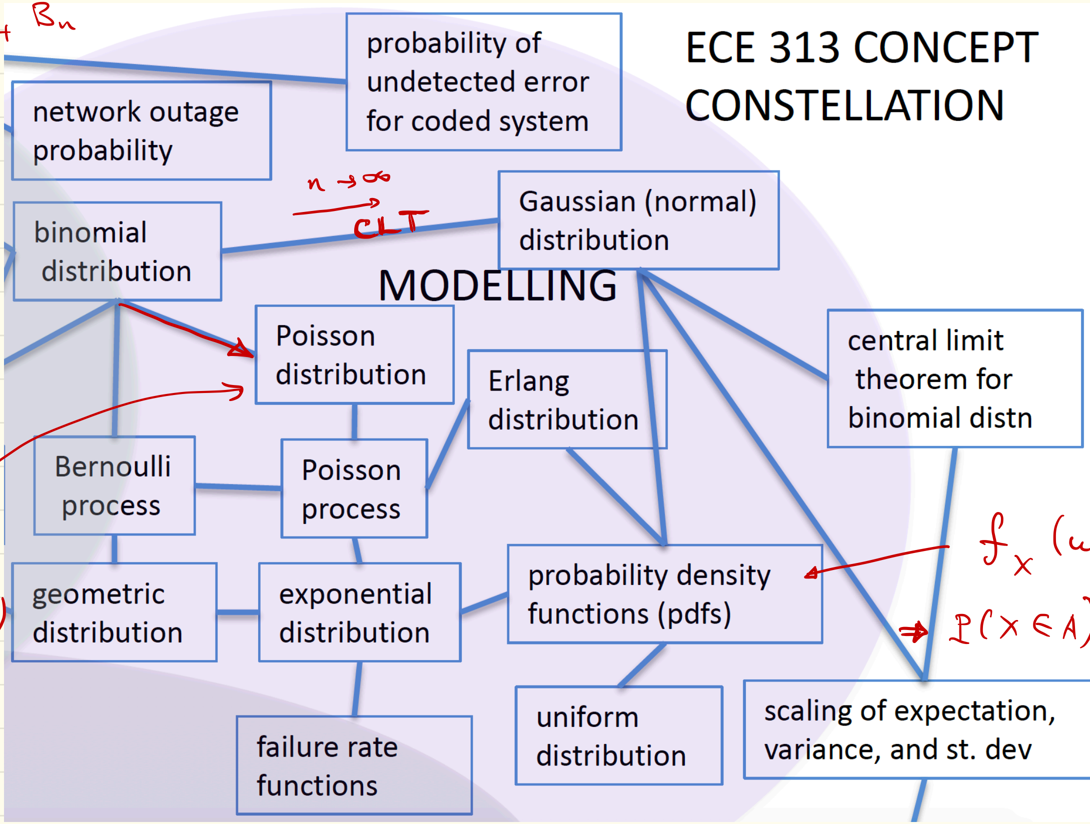
$X \sim \mathrm{Binom}(n,p) = B_1 + B_2 + \dots + B_n$

$P_X(k) = ?$

$E[X] =$

n big, $p \approx 0$

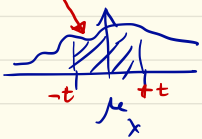
$\mathrm{Binom}(n,p) \to \mathrm{Pois}(\lambda = np)$



$f_X(u)$

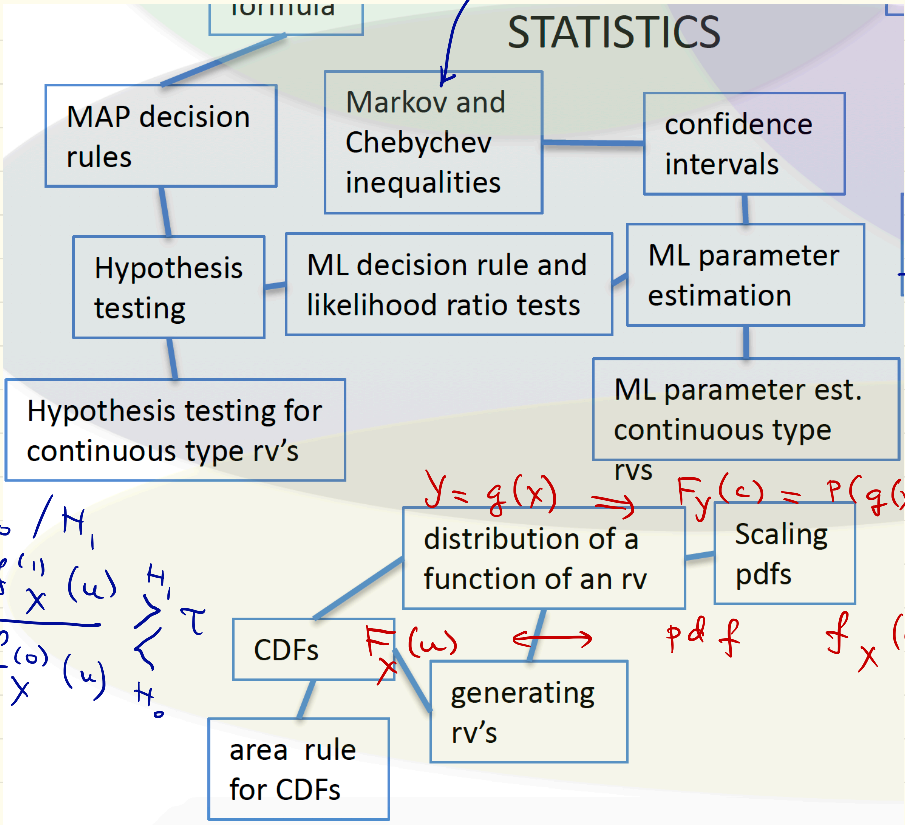
$\Rightarrow P(X \in A) = \int_{x \in A} f_X(u)\,du$

$$P(|X-\mu_X| < t) \geq \frac{\sigma_X^2}{t^2}$$



$H_0: \quad f_X^{(0)}(u)$

$H_1: \quad f_X^{(1)}(u)$



$f_X(x) = f(x;\theta)$

observe

$\theta^{(ML)} = \arg\max_\theta f(x;\theta)$

$u \to H_0 / H_1$

$$\Lambda(u) = \frac{f_X^{(1)}(u)}{f_X^{(0)}(u)} \underset{H_0}{\overset{H_1}{\gtrless}} \tau$$

$Y = g(X) \Rightarrow F_Y(c) = P(g(X) \leq c)$

$F_X(u) \longleftrightarrow$ pdf

$$f_X(u) = \frac{d}{du} F_X(u)$$

Use moments:

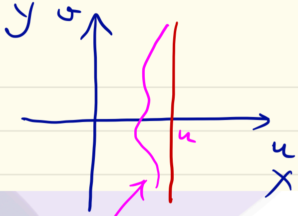
$E[X]$, $E[Y]$

$\mathrm{Var}[X]$, $\mathrm{Var}[Y]$

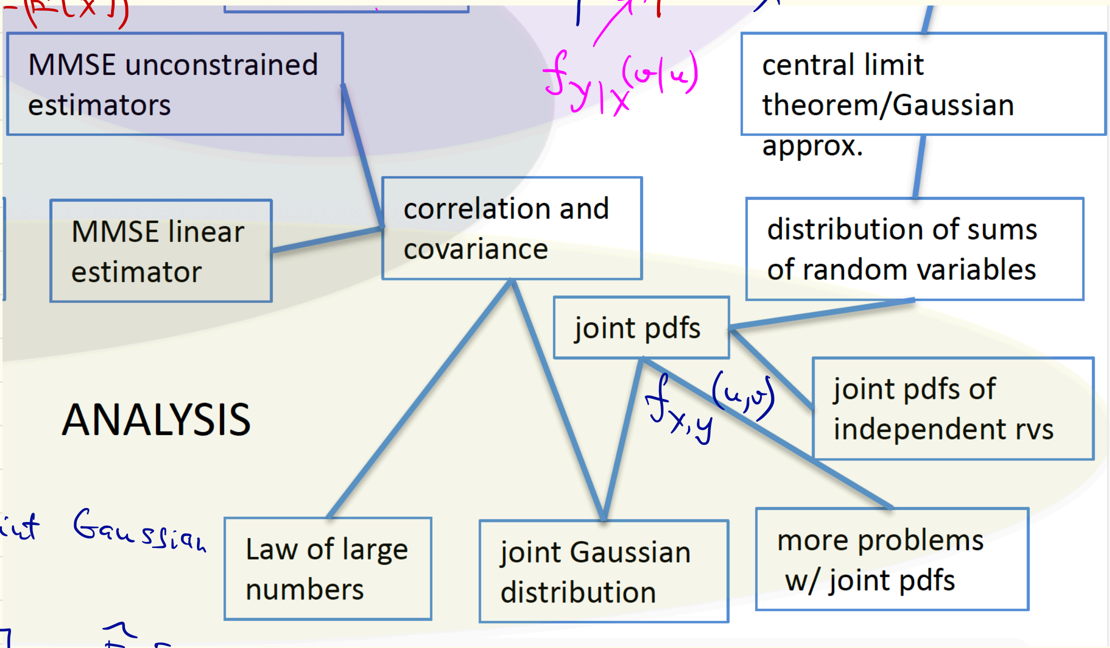
$\mathrm{Var}[X] = E[X^2] - (E[X])^2$

$\mathrm{Cov}(X,Y)$

Use distribution:



$f_{Y|X}(v|u)$



$f_{X,Y}(u,v)$

For joint Gaussian

$$E[Y|X] = \hat{E}[Y|X]$$

$$= E[Y] + \frac{\mathrm{Cov}(X,Y)}{\mathrm{Var}(X)}\left(X - E[X]\right)$$

5. **[3+3+4 points]** Consider two servers giving service to two tasks, in parallel. The service times of the first and second tasks are exponentially distributed with parameters λ_1 and λ_2, denoted by X_1 and X_2, respectively. The service times of the two tasks are independent from each other.

$X_1 \sim \mathrm{Exp}(\lambda_1)$

$X_2 \sim \mathrm{Exp}(\lambda_2)$

X_1 & X_2 are indep.

(a) What is $P(X_1 > 5 | X_1 > 3)$?

$$= P(X_1 > 2) = e^{-\lambda_1 \times 2}$$

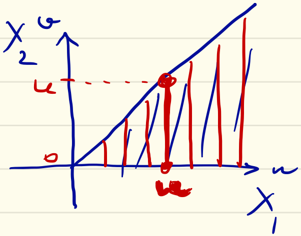
reset, wait for 2 more time units

(b) What is $\mathbb{E}[X_1 | X_1 > 3]$?

$$= E[X_1] + 3 = \frac{1}{\lambda_1} + 3$$

reset the start

(c) What is $P(X_1 > X_2)$?



$$f_{X_1, X_2}(u, v) = f_{X_1}(u) f_{X_2}(v)$$

$$P(X_1 > X_2) = \int_A f_{X_1, X_2} = \int_0^\infty \left(\int_0^u f_{X_2}(v)\, dv \right) f_{X_1}(u)\, du$$