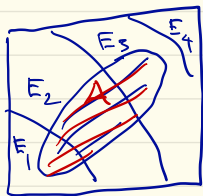


# ECE 313: Lecture 42: Review and Wrap-up



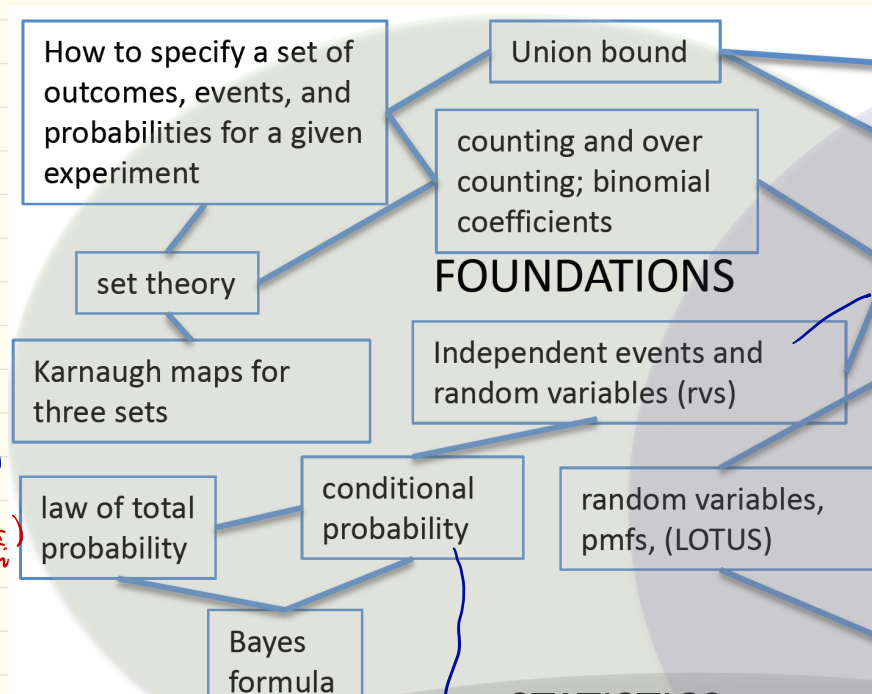
$$P(A) = \sum_i P(A|E_i)$$

$P(E_i) P(A|E_i)$

where

$$\bigcup E_i = \Omega$$

$$E_i \cap E_j = \emptyset$$



Independent

$$P(A|B) = P(A) P(B)$$

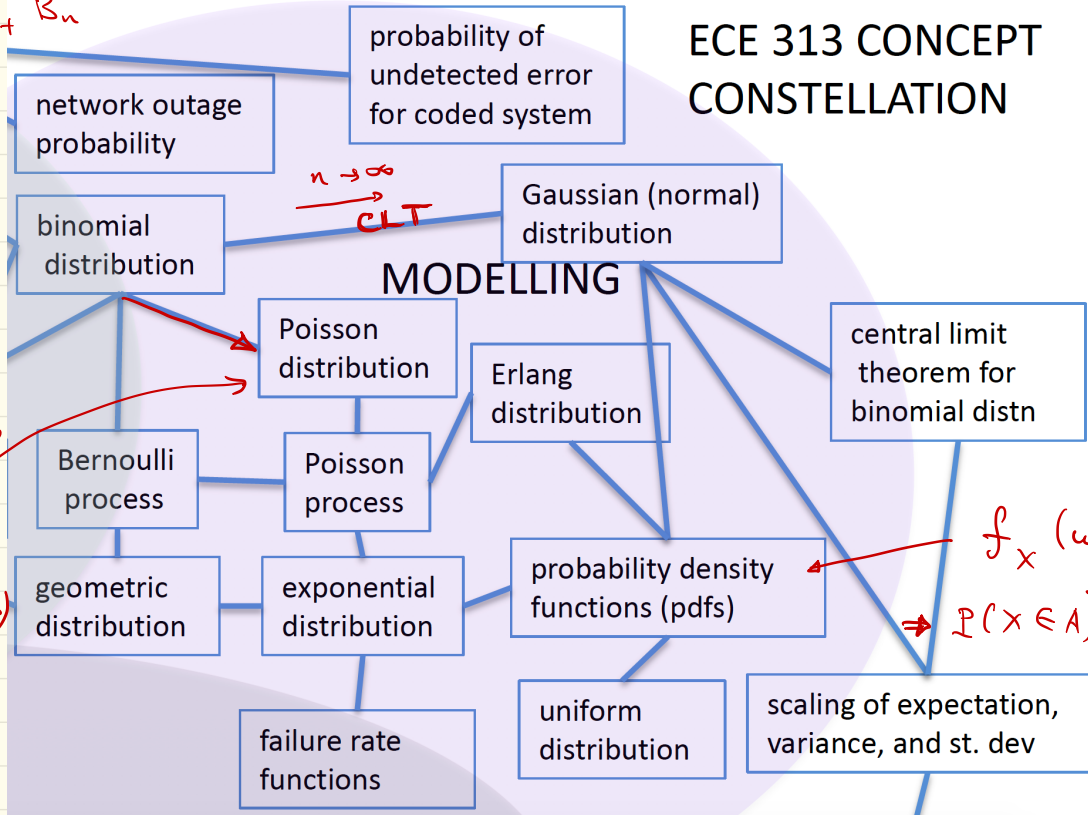
$$f_{X,Y}(u,v) = f_X(u) f_Y(v)$$

$$P(A|B) = P(A) P(B|A)$$

$$f_{X,Y}(u,v) = f_X(u) f_{Y|X}(v|u)$$

# ECE 313 CONCEPT CONSTELLATION

## MODELLING



probability of undetected error for coded system

network outage probability

binomial distribution

$n \rightarrow \infty$   
CLT

Gaussian (normal) distribution

Poisson distribution

Erlang distribution

central limit theorem for binomial distn

Bernoulli process

Poisson process

geometric distribution

exponential distribution

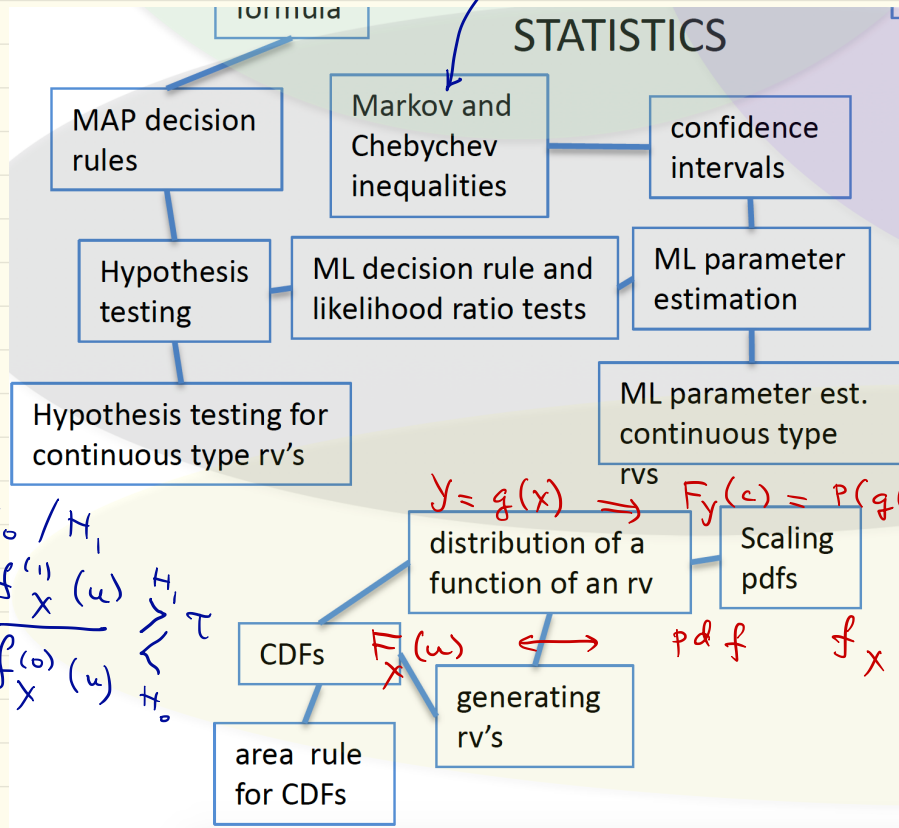
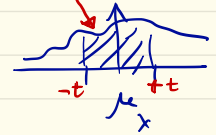
probability density functions (pdfs)

uniform distribution

scaling of expectation, variance, and st. dev

failure rate functions

$$P(|X - \mu_X| < t) \geq \frac{\sigma_X^2}{t^2}$$



$$H_0: f_X^{(0)}(u)$$

$$H_1: f_X^{(1)}(u)$$

$u \rightarrow$

$H_0 / H_1$

$$\Lambda(u) = \frac{f_X^{(1)}(u)}{f_X^{(0)}(u)} \gtrless_{H_0} \tau$$

$$Y = g(X) \Rightarrow F_Y(c) = P(g(X) \leq c)$$

$$f_X(u) = f(x; \theta)$$

observe

$$\hat{\theta}^{(ML)} = \arg \max_{\theta} f(x; \theta)$$

pdf

$$f_X(u) = \frac{d}{du} F_X(u)$$

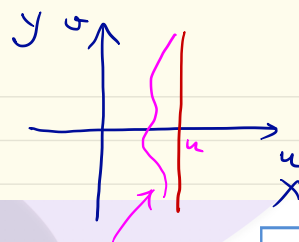
Use moments:

$$E[X], E[Y]$$

$$\text{Var}[X], \text{Var}[Y]$$

$$E[X^2] - (E[X])^2$$

$$\text{Cov}(X, Y)$$



Use distribution:

MMSE unconstrained estimators

MMSE linear estimator

correlation and covariance

central limit theorem/Gaussian approx.

distribution of sums of random variables

joint pdfs

joint pdfs of independent rvs

$$f_{X,Y}(u,v)$$

ANALYSIS

Law of large numbers

joint Gaussian distribution

more problems w/ joint pdfs

For joint Gaussian

$$E[Y|X] = \hat{E}[Y|X]$$

$$= E[Y] + \frac{\text{Cov}(X, Y)}{\text{Var}(X)} (X - E[X])$$

5. [3+3+4 points] Consider two servers giving service to two tasks, in parallel. The service times of the first and second tasks are exponentially distributed with parameters  $\lambda_1$  and  $\lambda_2$ , denoted by  $X_1$  and  $X_2$ , respectively. The service times of the two tasks are independent from each other.

$$X_1 \sim \text{Exp}(\lambda_1)$$

$$X_2 \sim \text{Exp}(\lambda_2)$$

$X_1$  &  $X_2$   
are indep.

- (a) What is  $P(X_1 > 5 | X_1 > 3)$ ?

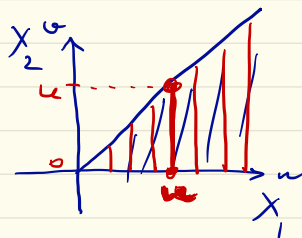
$$= P(\underbrace{X_1}_{\text{reset}} > \underbrace{2}_{\text{wait for 2 more time units}}) = e^{-\lambda_1 \times 2}$$

reset, wait for 2 more time units

- (b) What is  $\mathbb{E}[X_1 | X_1 > 3]$ ?

$$= \mathbb{E}[X_1] + 3 = \frac{1}{\lambda_1} + 3$$

↑  
reset the start



- (c) What is  $P(X_1 > X_2)$ ?

$$f_{X_1, X_2}(u, v) = f_{X_1}(u) f_{X_2}(v)$$

$$P(X_1 > X_2) = \int_A f_{X_1, X_2} = \int_0^{\infty} \left( \int_0^u f_{X_2}(v) dv \right) f_{X_1}(u) du$$