ECE 313: Lecture 3
Counting: principles and examples (cont.)
Binomial coefficients

Common setting for Binomial Coefficients

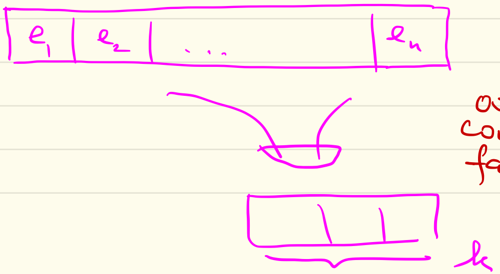
"How many subsets of k elements from a set of n elements?"

$\equiv$ Given a set of n elements $\quad (0 \le k \le n)$

Pick a subset of k elements.

How many ways?"

Binomial coef.



$$\binom{n}{k} = \binom{n}{n-k} = \frac{\overbrace{n\cdot(n-1)\cdots(n-k+1)}^{k \text{ terms}}}{k\,(k-1)\cdots 1} = \frac{n!}{k!\,(n-k)!}$$

over counting factor $\rightarrow$ $k\,(k-1)\cdots 1$

$\boxed{n! = 1\cdot 2\cdot 3 \cdots n}$

Ex: "ILLINI" 6-letters

How many 6-letter words can be formed from "ILLINI"

$I_1, I_2, I_3, L_1, L_2, N$

$\Rightarrow 6 \cdot 5 \cdot 4 \cdots 1 = 6!$ ways

Ex:

$I_1 \; L_1 \; L_2 \; I_2 \; N \; I_3 \equiv$ ___

2 3 → $2!$

1 4 6 → $3!$

$$\Rightarrow \frac{6!}{3!\,2!}$$

Ex: Number of Full House

$$\underbrace{\boxed{10_1}\,\boxed{10_2 \mid 10_3}}_{\#}\,\underbrace{\boxed{9_1 \mid 9_2}}$$

52 cards: Each $\boxed{N_F}$

$$N \in \{\overset{A}{1}, 2, 3, \dots, 10, \overset{J}{11}, \overset{Q}{12}, \overset{K}{13}\}$$

$$F \in \{1, 2, 3, 4\}$$

Full House = " 2 cards same number
& 3 cards same another number "

shuffle
$\Longleftrightarrow$

Ex:

$$\underbrace{\boxed{9_1 \mid 9_4}}_{\text{one number}}\quad \underbrace{\boxed{7_1 \mid 7_3 \mid 7_4}}_{\text{anther number}}$$

one number (w. 2 cards) anther number (w. 3 cards)

Method 1: $13 \cdot 12 \left(\binom{4}{2}\right)\binom{4}{3}$ with $\binom{4}{2} = \frac{4 \cdot 3}{1 \cdot 2}$

$$= 13 \cdot 12 \cdot \frac{\overset{2}{\cancel{4}} \cdot 3}{1 \cdot \cancel{2}} \cdot \frac{4 \cdot \cancel{3} \cdot \cancel{2}}{1 \cdot \cancel{2} \cdot \cancel{3}}$$

$$= 13 \cdot 12 \cdot 2 \cdot 3 \cdot 4$$

Method 2: $\dfrac{\overset{13 \cdot \cancel{4}\, 2}{\cancel{52}} \cdot 3}{1 \cdot \cancel{2}} \cdot \dfrac{\overset{12 \cdot 4}{\cancel{48}} \cdot \cancel{3} \cdot \cancel{2}}{\cancel{1 \cdot 2 \cdot 3}}$