ECE 313: Lecture 29
Joint CDFs and pdfs (cont.)

$$I_A(u,v) = \begin{cases} 1 & \text{if } (u,v) \in A \\ 0 & \text{else} \end{cases}$$

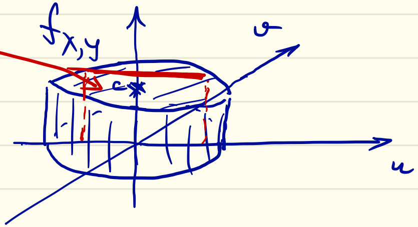
indicator function

Ex: pdf

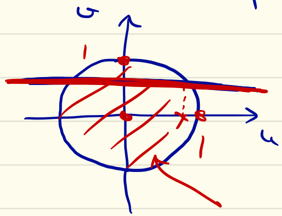
$$f_{X,Y}(u,v) = c\, I_{\underbrace{\{(u,v):\ u^2+v^2 \le 1\}}_{A}}$$

pdf (peanut butter!)

2D function



Top-down view



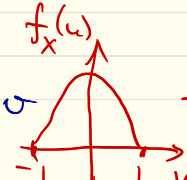
* Conditional pdfs:

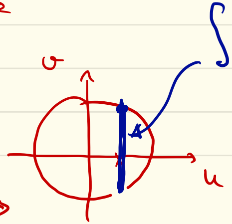
(Condition on $y = v_0$)

$$f_{X|Y}(u, v_0) = \frac{f_{X,Y}(u,v_0)}{f_Y(v_0)}$$

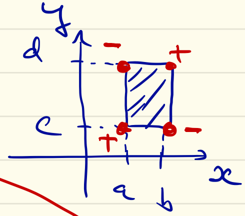
* Marginal pdfs:

$$f_X(u) = \int_{-\infty}^{+\infty} f_{X,Y}(u,v)\, dv$$





$$* \quad P((X,Y) \in A) = \iint_A f_{X,Y}(x,y)\, dx\, dy$$

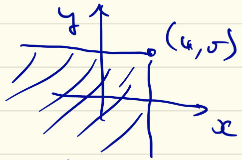


Special cases :

① $A = [a,b] \times [c,d]$

$$P((X,Y) \in A) = \int_c^d \int_a^b f_{X,Y}(x,y)\, dx\, dy$$

$$= \int_a^b dx \int_c^d dy\; f_{X,Y}(x,y)$$



② $A = \{ (x,y) : x \leq u,\ y \leq v \}$

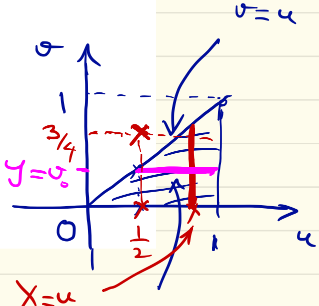
$$F_{X,Y}(u,v) = P\{ X \leq u,\ Y \leq v \} = \int_{-\infty}^{u} \int_{-\infty}^{v} f_{X,Y}(x,y)\, dy\, dx$$

Lemma : $P\{ a \leq X \leq b,\ c \leq Y \leq d \} = F_{X,Y}(b,d) - F_{X,Y}(a,d) - F_{X,Y}(b,c) + F_{X,Y}(a,c)$

5. **[8+8+4 points]** Let X and Y be continuous-type random variables with joint pdf

$$f_{X,Y}(u,v) = \begin{cases} 2, & 0 \le v \le u \le 1 \\ 0, & \text{else.} \end{cases}$$

Then the marginal pdf of Y is $f_Y(v_o) = \begin{cases} 2(1-v_o), & 0 \le v_o < 1 \\ 0, & \text{else.} \end{cases}$



$0 \le v \le u \le 1$

(a) Find $f_X(u)$, the marginal pdf of X.

$$f_X(u) = \int f_{X,Y}(u,v)\,dv = \int_0^u 2\,dv$$

$$= 2v\Big|_0^u = 2u$$

(b) Find $f_{X|Y}(u|v_o)$, the conditional pdf of X given Y.

for $v_o \le u \le 1$ $= \dfrac{f_{X,Y}(u,v_o)}{f_Y(v_o)} = \dfrac{\cancel{2}}{\cancel{2}(1-v_o)}$.

else $= 0$

(c) Are X and Y independent? Why?

$$\underbrace{f_{X,Y}(x,y)}_{=0} \overset{?}{=} \underbrace{f_X(x)}_{>0}\ \underbrace{f_Y(y)}_{>0}$$

NO for $(x,y) = \left(\frac{1}{2}, \frac{3}{4}\right)$