ECE 313: Lecture 27
Joint CDFs (Ch 4.1)
Joint pmfs (Ch 4.2)

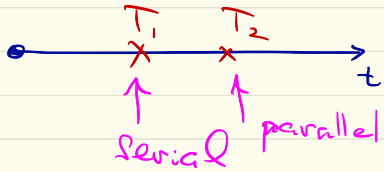
Ex: $T \sim \text{Exp}\{\lambda\}$

$$f_T(u) = \begin{cases} \lambda e^{-\lambda u} & u \geq 0 \\ 0 & \text{else} \end{cases}$$

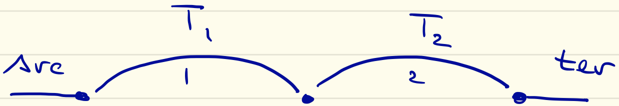
$$F_T(v) = 1 - e^{-\lambda v}$$

$$h_T(u) = \lambda$$

Review Failure Rate



Serial



Life time of whole network $T = \min\{T_1, T_2\}$

$$1 - F_T(t) = P\{T > t\} = P\{T_1 > t, T_2 > t\}$$

if T_1 & T_2 are ind.

$$= \underbrace{P\{T_1 > t\}}_{1 - F_{T_1}(t)} \cdot P\{T_2 > t\}$$

Parallel

$T = \max\{T_1, T_2\}$

$$P\{T > t\} = P\{(T_1 > t) \cup (T_2 > t)\}$$

$$= P\{T_1 > t\} + P\{T_2 > t\} - P\{T_1 > t, T_2 > t\}$$

Joint CDF $\qquad X \rightsquigarrow Y$

Before: often assume X & Y are independent

Now, more general X & Y are dependent

Most important: $\hat{Y} = \text{Predictor}_Y(X)$

Model X & Y jointly

$$(X, Y) \sim \underbrace{F_{X,Y}}_{\text{CDF}} \quad \text{or} \quad \underbrace{f_{X,Y}}_{\text{pdf}} \qquad \overbrace{p_{X,Y}}^{\text{pmf}}$$

Ex:

	$X=0$	$X=1$
$Y=1$	0.3	0.3
$Y=0$	0.1	0.3

$$p_{X,Y}(u, v) = P\{X = u, Y = v\}$$

$$u, v \in \{0, 1\}$$

T/H

$$P\{X = 0\} = P\{X=0, Y=0\} + P\{X=0, Y=1\}$$

1. **[8+6+10+6 points]** The joint pmf of two discrete-type random variables is as shown in the table below

$\rightarrow \sum$ rows

$p_{X,Y}(2,2) =$
$P\{X=2, Y=2\} = 0.25$

	$X=1$	$X=2$	$X=3$	$X=4$	p_Y
$Y=1$	0.2	0	0.1	0.1	0.4
$Y=2$	0	0.25	0.15	0.05	0.45
$Y=3$	0.05	0	0	0.1	0.15
$\sum$ cols $\downarrow$	0.25	0.25	0.25	0.25	

Total law of prob.

(a) Find the marginal pmfs of X and Y.

$$p_X(u) = P\{X=u\} = \sum_v P\{X=u, Y=v\}$$
$$= \sum_v p_{X,Y}(u,v)$$

$$p_Y(v) = \sum_u p_{X,Y}(u)$$

(b) Are X and Y independent? Justify your answer.

$\Updownarrow$

$$p_{X,Y}(u,v) = p_X(u)\, p_Y(v) \quad \text{for all } u,v$$

No: $0 = p_{X,Y}(2,1) \neq p_X(2) \cdot p_Y(1)$

(c) Find the conditional pmf $p_{Y|X}(v|4)$.

$$p_{Y|X}(v|u) = P\{Y=v \mid X=u\} = \frac{P\{Y=v, X=u\}}{P\{X=u\}} = \frac{p_{X,Y}(u,v)}{p_X(u)}$$

$$p_{Y|X}(v|4) = \frac{(0.1;\ 0.05;\ 0.1)}{0.25} \quad v = 1, 2, 3$$

(d) Find $P\{X < Y\}$.

$= 0.05$