**ECE 313: Lecture 26**
**Failure rate functions (Ch 3.9)**
**Binary hypothesis testing for continuous type random variables (Ch 3.10)**

Key point of continuous-type r.v.

pdf : $f_X(x) \longrightarrow \underbrace{P\left\{x-\frac{\varepsilon}{2} \leq X \leq x+\frac{\varepsilon}{2}\right\}}_{\text{interval of length } \varepsilon} \approx \varepsilon \cdot f_X(x)$

$$f_X(x) = \lim_{\varepsilon \to 0} \frac{P\left\{x-\frac{\varepsilon}{2} \leq X \leq x+\frac{\varepsilon}{2}\right\}}{\varepsilon} \longleftarrow$$

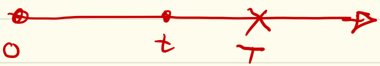
$$\| $$

$$F_X\left(x+\frac{\varepsilon}{2}\right) - F_X\left(x-\frac{\varepsilon}{2}\right)$$

( also $F_X'(x) = f_X(x)$ )

⊛ Failure rate : $T$ denotes lifetime of a device

Failure rate $\quad h(t) \stackrel{\text{def}}{=} \lim_{\varepsilon \to 0} \frac{P(t < T \leq t+\varepsilon \mid T > t)}{\varepsilon}$




$$= \lim_{\varepsilon \to 0} \frac{P(t < T \leq t+\varepsilon,\ T > t)}{\varepsilon \cdot P(T > t)} = \frac{f_T(t)}{1 - F_T(t)}$$

Ex: $T \sim \text{Exp}(\lambda)$

Recall:
$$\begin{cases} f_T(t) = \begin{cases} \lambda e^{-\lambda t} & t \geq 0 \\ 0 & \text{else} \end{cases} \\ F_T(t) = 1 - e^{-\lambda t} \end{cases}$$

$$\Rightarrow P(T > t) = 1 - F_T(t) = e^{-\lambda t}$$

In this case:
$$h(t) = \frac{f_T(t)}{1 - F_T(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

for $t > 0$

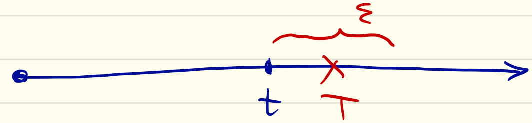
Interpretation: At time $t$, the prob that the device would fail in the next $\varepsilon$ (time unit) is $h(t) \cdot \varepsilon = \lambda \cdot \varepsilon$

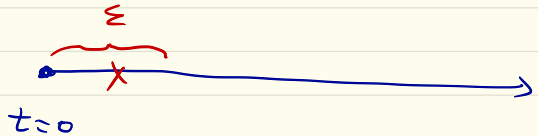
Relate to memoryless property of exponential r.v. $T$

$$P(t < T \leq t+\varepsilon \mid T > t) = \frac{P(T > t,\ T \leq t+\varepsilon)}{P(T > t)}$$

$$= \frac{F_T(t+\varepsilon) - F_T(t)}{e^{-\lambda t}} = \frac{-e^{-\lambda(t+\varepsilon)} + e^{-\lambda t}}{e^{-\lambda t}} = 1 - e^{-\lambda \varepsilon}$$

Hence $\quad \mathbb{P}(t < T \leq t+\varepsilon \mid T > t) = \mathbb{P}(T < \varepsilon)$

$P$ 


$= P$ 


Typical devices :


Given failure rate $h(t)$ of $T$

$$\Rightarrow \quad F_T(t) = 1 - e^{-\int_0^t h(s)\, ds}$$

## ⊛ Binary hypothesis testing

**discrete type : pmf**

| | | $k$: 0 | 1 | 2 |
|---|---|---|---|---|
| $H_1$ : | $p_1(X=k)$ | 0.1 | 0.2 | 0.7 |
| $H_0$ : | $p_0(X=k)$ | 0.6 | 0.3 | 0.1 |
| likelihood ratio : | $\Lambda(k) = \dfrac{p_1(k)}{p_0(k)}$ | $\frac{1}{6}$ | $\frac{2}{3}$ | 7 |

**Continuous type : pdf**

$$f_{X,1}(x)$$

$$f_{X,0}(x)$$

$$\Lambda(x) = \frac{f_1(x)}{f_0(x)}$$

Decision rule:

ML :

$$\Lambda(x) \underset{H_0}{\overset{H_1}{\gtrless}} 1$$

MAP :

$$\Lambda(x) \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\pi_0}{\pi_1}$$

$\pi_0 \leftarrow P(H_0)$

$\pi_1 \leftarrow P(H_1)$