ECE 313: Lecture 2
Probability space and axioms of probability
Set theory: de Morgan's law, Karnaugh maps
Counting: principles and examples

Probability Space $= (\Omega, \mathcal{F}, P)$

(i) Ω : set of all outcomes

(ii) $\mathcal{F}$: set of all events → each is a subset of Ω

(iii) P : prob. measure on each $A \in \mathcal{F}$

Ex:

B dice

one element of the set

$$\Omega = \{ (i, j) : 1 \le i \le 6,\ 1 \le j \le 6 \}$$

W dice

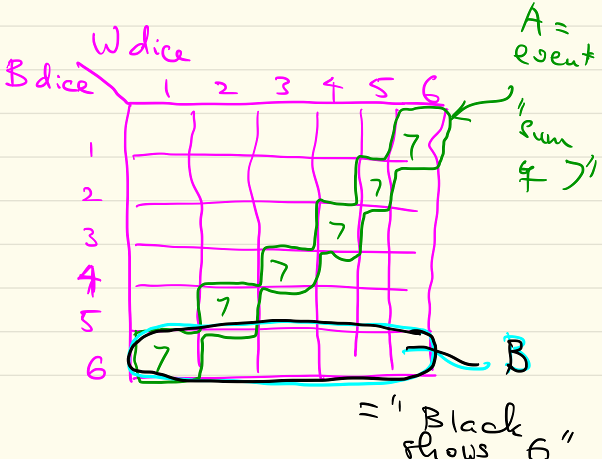
$|\Omega|$ = "cardinity of Ω"

$= 36$

$A = \{ (1,6), (2,5), \ldots, (6,1) \}$

$|A| = 6$

$$P(A) = \frac{|A|}{|\Omega|} = \frac{6}{36}$$



Ex: Throw 2 dices B & W

$\Omega = \{ (i,j) : 1 \leq i,j \leq 6 \}$ $\quad |\Omega| = 6 \times 6 = 36$

$\mathcal{F} = \{ \text{all subsets of } \Omega \} = \{ A : A \subset \Omega \}$

$|\mathcal{F}| = 2^{|\Omega|} = 2^{36}$

⊛ Set operations

Let A, B are 2 sets

Intersection: $AB = \{ c : c \in A \text{ and } c \in B \}$

Union $A \cup B = \{ d : d \in A \text{ or } d \in B \}$

Ex: A = " sum of 7 " $\quad AB = \{ (6,1) \}$

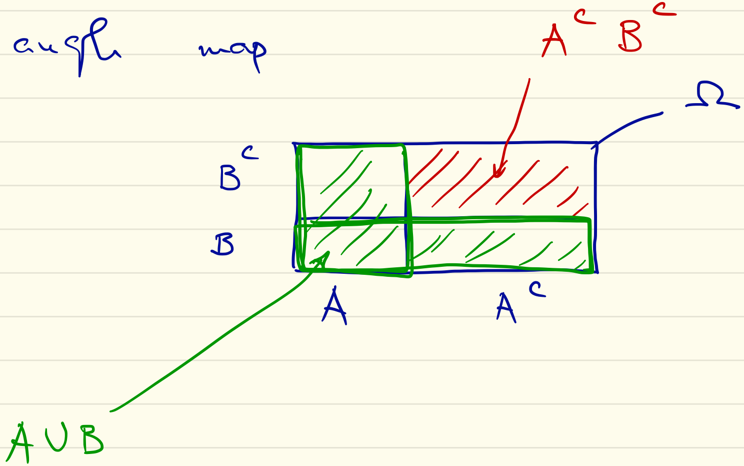
B = " B' of dice show 6 "

$(A \cup B) = \{ (1,6), (2,5), \ldots, (6,1), (6,2), \ldots, (6,6) \}$

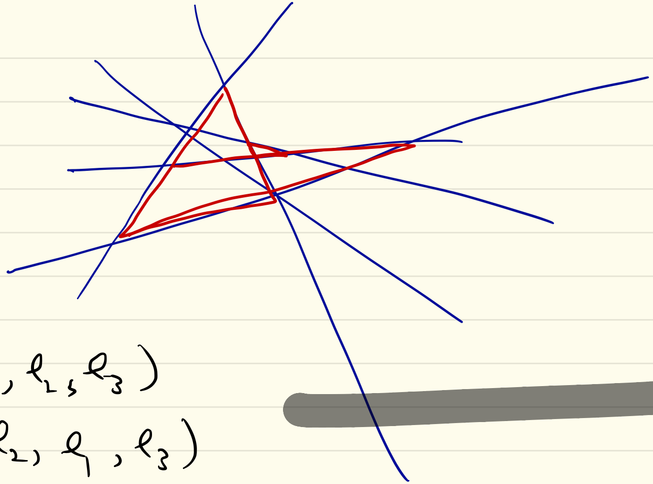
Complementary of a set A

$A^c = \{ b : \quad b \in \Omega \text{ and } \underbrace{b \notin A}_{\uparrow \text{ not in}} \}$

⊛ Karnaugh map



Hence $(A^c B^c)^c = A \cup B$ (de Morgan



6 lines

How many triangles?

one triangle = 3 lines

$$= (\ell_1, \ell_2, \ell_3)$$

$$(\ell_1, \ell_2, \ell_3) \equiv (\ell_2, \ell_1, \ell_3) \equiv (\ell_3, \ell_2, \ell_1)$$

$$\binom{6}{3} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1}$$

$$\binom{n}{k} = \frac{n(n-1)\ldots(n-k+1)}{k(k-1)\ldots(2)(1)} = \frac{n(n-1)\ldots 1}{k(k-1)\ldots 1\,(n-k)\ldots 1}$$

$$= \frac{n!}{k!\,(n-k)!}$$