ECE 313: Lecture 17
Error probablity and optimality of the MAP decision rule
Network outage probablity and applications of the union bound

MAP

⇓

Likehood matrix: $P(\{X=k\} | H_i)$

Joint prob matrix

$P(\{X=k\}, H_i) = P(H_i) P(\{X=k\} | H_i)$

	X=1	X=2	X=3	X=4
H_1	0.1	0.2	0.3	0.4
H_0	0.4	0.3	0.2	0.1

$\times \pi_1 = 0.8 \Rightarrow$

$\times \pi_0 = 0.2 \Rightarrow$

	X=1	X=2	X=3	X=4
H_1	0.08	0.16	0.24	0.32
H_0	0.08	0.06	0.04	0.02

Error: $P_{false\text{-}alarm} = P(\text{Declare } H_1 | H_0)$

$P_{miss} = P(\text{Declare } H_0 | H_1)$

$$P_{error} = \underbrace{P(H_0)}_{\pi_0} \cdot P_{f.a.} + \underbrace{P(H_1)}_{\pi_1} P_{miss}$$

= sum of NOT underscored entries in joint prob matrix

Hence MAP decision minimize the P_{error}

Decision rule :

$$\Lambda(k) \underset{H_0}{\overset{H_1}{\gtrless}} \tau \quad \left(\begin{array}{ll} = 1 & \text{ML} \\ = \frac{\pi_0}{\pi_1} & \text{MAP} \end{array} \right)$$

One particular tie breaking

$$\Lambda(k) \geq \tau \longrightarrow H_1$$

$$\Lambda(k) < \tau \longrightarrow H_0$$

Ex :

H_0 : $X \sim \text{Poisson}(\lambda_0)$

H_1 : $X \sim \text{Poisson}(\lambda_1)$

$(X = k) \xrightarrow{\text{Decision}} H_0 / H_1$

$$\Lambda(k) = \frac{P(\{X=k\} \mid H_1)}{P(\{X=k\} \mid H_0)} = \frac{\lambda_1^k e^{-\lambda_1} / k!}{\lambda_0^k e^{-\lambda_0} / k!}$$

$$= \left(\frac{\lambda_1}{\lambda_0}\right)^k e^{\lambda_0 - \lambda_1} \underset{H_0}{\overset{H_1}{\gtrless}} \tau$$

Union bound:

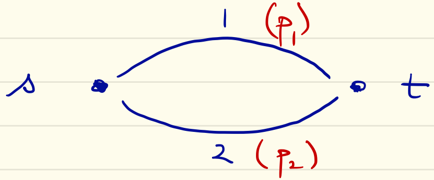
$$F = F_1 \cup F_2 \cup \dots \cup F_n$$

$$P(F) = P(F_1 \cup F_2 \cup \dots \cup F_n)$$
$$\leq P(F_1) + P(F_2) + \dots + P(F_n)$$

Network Outage

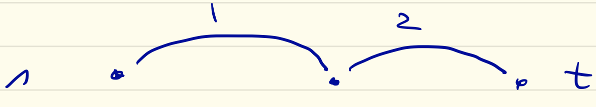
Notation: link k has failure prob $P(F_k) = p_k = p \approx 0$

(1)



$$P(F) = P(F_1 F_2)$$
$$= P(F_1)\, P(F_2)$$
$$= p_1 p_2$$

(2)



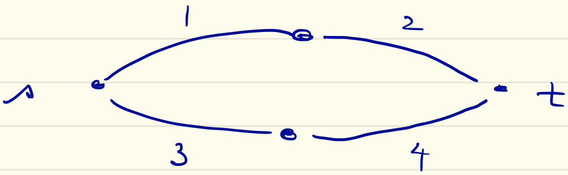
$$P(F) = P(F_1 \cup F_2)$$
$$= P(F_1) + P(F_2) - P(F_1 F_2)$$
$$= p_1 + p_2 - p_1 p_2$$

→ Union bound $\leq 2p$

Alternatively,

$$P(F) = 1 - P(F^c) = 1 - P(F_1^c)\, P(F_2^c) = 1 - (1-p_1)(1-p_2)$$

(3)



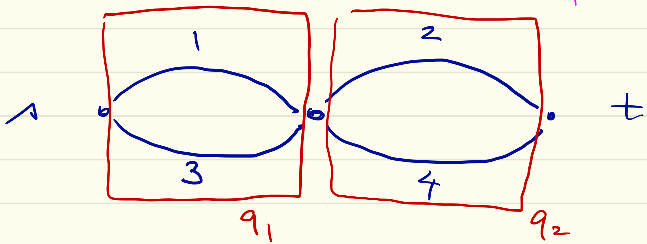
$$P(F) = P(\text{link } 1\text{-}2 \text{ fail}) \; P(\text{link } 3\text{-}4 \text{ fail})$$

$$= P(F_1 \cup F_2) \; P(F_3 \cup F_4)$$

$$= (p_1 + p_2 - p_1 p_2) \; (p_3 + p_4 - p_3 p_4)$$

Union bound $\leq 2p \cdot 2p = 4p^2$

(4)



$$P(F) = P(Q_1 \cup Q_2)$$

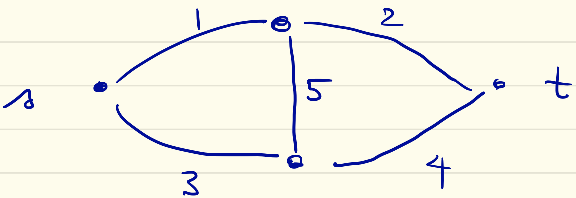
$$= q_1 + q_2 - q_1 q_2$$

$q_1 = p_1 p_3$

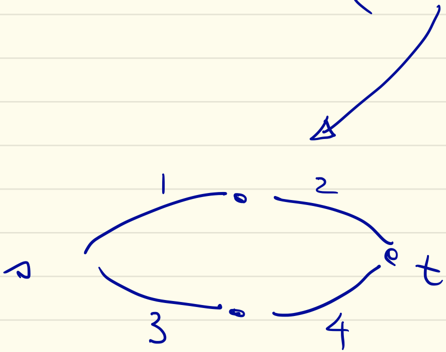
$q_2 = p_2 p_4$

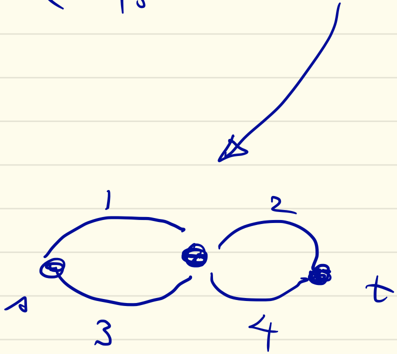
Union bound $\leq 2q = 2p^2$

(5)



$$P(F) = p_5 \cdot \{ \quad \} + (1 - p_5) \cdot \{ \quad \}$$





Union bound $\leq p \cdot 4p^2 + (1-p)\, 2p^2 = 2p^2 + 2p^3$