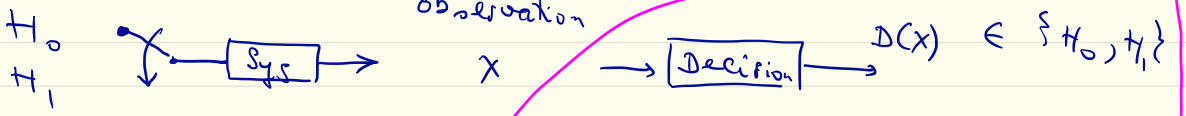
ECE 313: Lecture 16
MAP decision rules
Union bound and its application

$P_{fa} = P(\text{Declare } H_1 \mid H_0) = \sum$ on H_0 row

$P_{miss} = P(\text{Declare } H_0 \mid H_1) = \sum$ but NOT underscore



Likelihood table

$p_i(k) = P(\{X = k\} \mid H_i)$

	$X=1$	$X=2$	$X=3$	$X=4$
H_1	0.1	0.2	0.3	0.4
H_0	0.4	0.3	0.2	0.1
$\Lambda(k) = \frac{p_1(k)}{p_0(k)}$	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{3}{2}$	4

$\Lambda(k) \underset{H_0}{\overset{H_1}{\gtrless}} \tau \qquad \left(\overset{M-L:}{\tau = 1}\right)$

Prior knowledge: $P(H_i) = \pi_i \quad (i = 0, 1)$

$P(\{X = k\}, H_i) = P(H_i)\, P(\{X = k\} \mid H_i)$

$= \pi_i\, p_i(k)$ — joint prob

$\pi_0 = 0.2, \quad \pi_1 = 0.8$

	$X=1$	$X=2$	$X=3$	$X=4$
H_1	0.08	0.16	0.24	0.32
H_0	0.08	0.06	0.04	0.02

ML: given $X = k$ ✓ pick H_i $(i \in \{0,1\})$ that

$$\max_{i \in \{0,1\}} P(\{X = k\} \mid H_i)$$

MAP (Maximum a posterior) decision:

given $X = k$ pick H_i that

$$\max_{i \in \{0,1\}} P(H_i \mid \{X = k\})$$

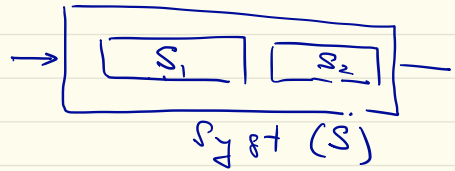
$$= \max_{i \in \{0,1\}} \frac{P(\{X = k\}, H_i)}{P(\{X = k\})}$$

$$\Leftrightarrow \quad \underbrace{\Pi_1 p_1(k)}_{P(\{X=k\}, H_1)} \gtrless_{H_0}^{H_1} \underbrace{\Pi_0 p_0(k)}_{P(\{X=k\}, H_0)}$$

$$\Leftrightarrow \quad \Lambda(k) = \frac{p_1(k)}{p_0(k)} \gtrless_{H_0}^{H_1} \frac{\Pi_0}{\Pi_1} \leftarrow \tau$$

Union bound :

Context : Estimate the prob of system failure



S fails if either S_1 fails or S_2 fails.

A B

$$P\{S \text{ fails}\} = P(A \cup B)$$

$$= P(A) + P(B) - P(A \cap B)$$

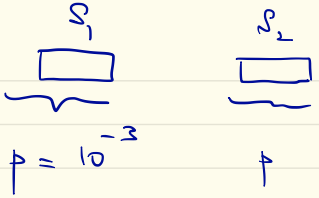
$P(A \cap B) \approx 0$ if $P(A)$ & $P(B)$ small

$$\leq P(A) + P(B)$$

union bound

$$\left(\text{or } P(A \cup B) \right)$$

Ex:



$P(\text{whole syst fails}) = P(A \cup B)$

if one sub sys fails

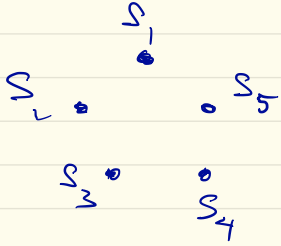
$= P(A) + P(B) - P(A \cap B)$

$P(A) \cdot P(B)$

$= p + p - p^2$

$\leq 2p$ ← union bound

Ex:



whole syst fails if at least 2 sub sys fails

$P(\text{whole sys fails}) = P(F_1F_2 \cup F_1F_3 \cup F_1F_4 \cup \dots \cup F_4F_5)$

$\overset{5.4}{\underset{2.1}{\leq}} \binom{5}{2} p^2 = P(F_iF_j) = P(F_i)P(F_j)$

$= 10\,p^2$