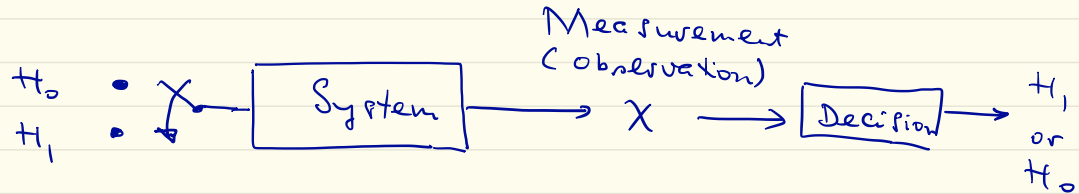


ECE 313: Lecture 15

Hypothesis testing -- probabilities of false alarm and miss

ML decision rule and likelihood ratio tests

MAP decision rules



Ex :

H_0 : no cancer
 H_1 : cancer

X : PSA level

Decision $D(x) = \begin{matrix} H_0 \\ H_1 \end{matrix}$

Error :

1. H_0

→

Decision $D(x) = H_1$ ← false alarm

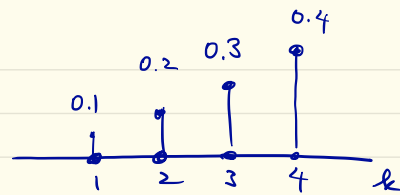
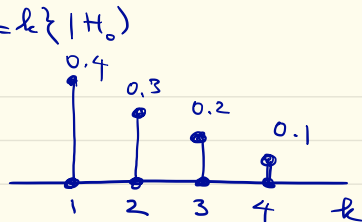
2. H_1

→

$D(x) = H_0$ ← miss

$$P_0(k) = P(\{X=k\} | H_0)$$

E_X : pmf:



X: course grade

H_0 : "bad" \leftrightarrow no hire

H_1 : "good" \leftrightarrow hire

A = 4

B = 3

C = 2

D = 1

$$P(\{X=k\} | H_i) = P_i(k)$$

| | X=1 | X=2 | X=3 | X=4 |
|--------------|---------------|---------------|---------------|-----|
| H_1 | 0.1 | 0.2 | 0.3 | 0.4 |
| H_0 | 0.4 | 0.3 | 0.2 | 0.1 |
| $\Lambda(k)$ | $\frac{1}{4}$ | $\frac{2}{3}$ | $\frac{3}{2}$ | |

Σ not under score entries in row H_0

Decision: $\{X=k\} \rightarrow H_0$ or H_1

E_X :

$X \in \{3, 4\} \rightarrow H_1$ declare

else $\rightarrow H_0$

$$P_{\text{false-alarm}} = P(\text{Declare } H_1 | H_0) =$$

$$0.2 + 0.1 = 0.3$$

\uparrow miss

$$= P(\text{Declare } H_0 | H_1) =$$

$$0.1 + 0.2 = 0.3$$

One decision rule: Maximum Likelihood

$X = k \rightarrow$ Declare H_0 if $p_0(k) > p_1(k)$
or H_1 else $p_0(k) < p_1(k)$

Define likelihood ratio:

$$\Lambda(k) = \frac{p_1(k)}{p_0(k)}$$

$$\Lambda(k) \underset{H_0}{\overset{H_1}{>}} 1$$

Extension: $\Lambda(k) \underset{H_0}{\overset{H_1}{>}} \tau$ (threshold) > 0

Note: By varying τ we obtain different decision rules (decision boundaries)