Homework 4
Date due: Monday, March 1, 2021

1. **Place** each of the following LPs into *standard form* and **define** the corresponding $A$, $b$ and $c$ arrays

   (a) \[
   \begin{align*}
   \text{max} & \quad 45x_1 + 15x_3 \\
   \text{s.t.} & \quad 4x_1 - 2x_2 + 9x_3 = 22 \\
   & \quad -2x_1 + 5x_2 - x_3 \geq 1 \\
   & \quad x_1 - x_2 \leq 5 \\
   & \quad x_1, x_2, x_3 \geq 0
   \end{align*}
   \]

   (b) \[
   \begin{align*}
   \text{max} & \quad 15(x_1 + 2x_2) + 11(x_2 - x_3) \\
   \text{s.t.} & \quad 3x_1 \geq x_1 + x_2 + x_3 \\
   & \quad 0 \leq x_j \leq 3 \quad j = 1, 2, 3
   \end{align*}
   \]

2. The following plot shows several feasible points in a linear program and contours of its objective function.
(a) Solve the problem graphically

(b) Determine whether the following sequence of solutions P1, P7, and P6 could have been generated by the application of the simplex algorithm to the corresponding LP problem in standard form. Provide the rationale for your answer.

3. Problem 2.30 in Ravindran, p.67
4. Problem 2.31 in Ravindran, p.68
5. Problem 2.32 in Ravindran, p.68

6. Consider the linear program

\[
\begin{align*}
\text{max} & \quad 4y_1 + 5y_2 \\
\text{s.t.} & \quad -y_1 + y_2 \leq 4 \\
& \quad y_1 - y_2 \leq 10 \\
& \quad y_1, y_2 \geq 0
\end{align*}
\]

(a) Show graphically that the model is unbounded.

(b) Add slacks \( y_3 \) and \( y_4 \) to place the model in standard form.

(c) Starting with an all slacks basic feasible solution, apply simplex algorithm to establish that the original model is unbounded.

7. Consider the LP

\[
\begin{align*}
\text{max} & \quad x_1 \\
\text{s.t.} & \quad 6x_1 + 3x_2 \leq 18 \\
& \quad 12x_1 - 3x_2 \leq 0 \\
& \quad x_1, x_2 \geq 0
\end{align*}
\]

(a) Solve the problem graphically.

(b) Add slack variables to place the model in standard form.

(c) Apply the simplex algorithm to compute an optimal solution starting with an all-slack-variables basic feasible solution.

8. The NCAA is making plans for distributing tickets to the upcoming regional basketball championships. The up to 10,000 available seats will be divided between the media, the competing universities, and the general public. Media people are admitted free, but the NCAA receives 45 \$ per ticket from universities and 100 \$ per
ticket from the general public. At least 500 tickets must be reserved for the media, and at least half as many tickets should go to the competing universities as to the general public. Within these restrictions, the NCAA wishes to find the allocation that raises the most money. An optimization is given in the table.

<table>
<thead>
<tr>
<th>Name</th>
<th>optimal value</th>
<th>basic non-basic</th>
<th>lower bound</th>
<th>upper bound</th>
<th>objective coefficient</th>
<th>reduced objective</th>
<th>lower range</th>
<th>upper range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>500.00</td>
<td>basic</td>
<td>0.000</td>
<td>$\infty$</td>
<td>0.000</td>
<td>0.000</td>
<td>-$\infty$</td>
<td>81.667</td>
</tr>
<tr>
<td>$x_2$</td>
<td>3166.667</td>
<td>basic</td>
<td>0.000</td>
<td>$\infty$</td>
<td>45.000</td>
<td>0.000</td>
<td>-200.000</td>
<td>100.00</td>
</tr>
<tr>
<td>$x_3$</td>
<td>6333.333</td>
<td>basic</td>
<td>0.000</td>
<td>$\infty$</td>
<td>100.000</td>
<td>0.000</td>
<td>45.000</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Name</th>
<th>type</th>
<th>optimal dual</th>
<th>r.h.s. coefficient</th>
<th>slack</th>
<th>lower range</th>
<th>upper range</th>
</tr>
</thead>
<tbody>
<tr>
<td>seats</td>
<td>$\leq$</td>
<td>81.667</td>
<td>10000.000</td>
<td>0.000</td>
<td>500.000</td>
<td>$\infty$</td>
</tr>
<tr>
<td>other</td>
<td>$\geq$</td>
<td>-36.667</td>
<td>0.000</td>
<td>-0.000</td>
<td>-4750.000</td>
<td>9500.00</td>
</tr>
<tr>
<td>media</td>
<td>$\geq$</td>
<td>-81.667</td>
<td>500.000</td>
<td>-0.000</td>
<td>0.000</td>
<td>10000.000</td>
</tr>
</tbody>
</table>

(a) **Formulate** this problem as an LP. **Explain** each decision variable, **identify** the coefficients in the objective function and **state** each constraint.

(b) **Interpret** the left-hand-side coefficients of the decision variables in the constraints

9. Consider the statement in the problem 8 above. **Answer** the following questions

(a) What is the marginal cost to the NCAA of each seat guaranteed for the media

(b) Suppose that there is an alternative arrangement of the dome where the games will be played that can provide 15,000 seats. How much additional revenue would be gained from the expanded seating? How much would it be for 20,000 seats?

(c) Since television revenue provides most of the income for NCAA events, another proposal reduce the price of general public tickets to $50. How much revenue would be lost from this change? What if the price were $30?
(d) Media-hating coach Sobby Day wants the NCAA to restrict media seats to 20% of those allocated for universities. Could this policy change the optimal solution? How about 10%?

(e) To accommodate high demand from student supporters of participating universities, the NCAA is considering marketing a new “scrunch seat” that consumes only 80% of a regular bleacher seat but counts fully against the “university ≥ half public” rule. Could an optimal solution allocate any such seats at a ticket price of $35? At a price of $25?

10. Professor Proof is trying to arrange for the implementation in a computer program of his latest operations research algorithm. He can contract with any mix of three sources for help: unlimited hours from undergraduates at 4 $/hour; up to 500 hours of

<table>
<thead>
<tr>
<th>name</th>
<th>optimal value</th>
<th>basic/non-basic</th>
<th>lower bound</th>
<th>upper bound</th>
<th>objective coefficient</th>
<th>reduced objective</th>
<th>lower range</th>
<th>upper range</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_1)</td>
<td>82.353</td>
<td>basic</td>
<td>0.000</td>
<td>(\infty)</td>
<td>4.000</td>
<td>0.000</td>
<td>-4.444</td>
<td>5.000</td>
</tr>
<tr>
<td>(x_2)</td>
<td>0.000</td>
<td>non-basic</td>
<td>0.000</td>
<td>5000</td>
<td>10.000</td>
<td>2.235</td>
<td>7.765</td>
<td>(\infty)</td>
</tr>
<tr>
<td>(x_3)</td>
<td>983.529</td>
<td>basic</td>
<td>0.000</td>
<td>(\infty)</td>
<td>25.000</td>
<td>0.000</td>
<td>20.000</td>
<td>31.333</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>name</th>
<th>type</th>
<th>optimal dual</th>
<th>rhs coefficient</th>
<th>slack</th>
<th>lower range</th>
<th>upper range</th>
</tr>
</thead>
<tbody>
<tr>
<td>professional</td>
<td>(\geq)</td>
<td>25.882</td>
<td>1000.000</td>
<td>-0.000</td>
<td>164.000</td>
<td>1093.333</td>
</tr>
<tr>
<td>proof</td>
<td>(\leq)</td>
<td>-5.882</td>
<td>164.000</td>
<td>0.000</td>
<td>150.000</td>
<td>1000.000</td>
</tr>
<tr>
<td>graduate</td>
<td>(\leq)</td>
<td>0.000</td>
<td>500.000</td>
<td>500.000</td>
<td>0.000</td>
<td>(\infty)</td>
</tr>
</tbody>
</table>

graduate students at the rate of 10 $/hour, or unlimited help from professional programmers at 25 $/hour. The full project would take a professional at least 1000 hours but graduate students are only 0.3 as productive, and undergraduates, 0.2. Proof only has 164 hours of his own time to devote to the effort, and he knows from experience that undergraduate programmers require more supervision than
graduate students, who, in turn, require more than professionals. In particular, he estimates that he will have to invest 0.2 hours of his own time per hour of undergraduate programming, 0.1 hour of his time per hour of graduate programming and 0.05 hour of his time per hour of professional programming. The table above summarizes the optimization output.

(a) Briefly explain how this problem can be modeled by the following LP

\[
\begin{align*}
\text{min} & \quad 4x_1 + 10x_2 + 25x_3 \\
\text{s.t.} & \quad 0.2x_1 + 0.3x_2 + x_3 \geq 1,000 \\
& \quad 0.2x_1 + 0.1x_2 + 0.05x_3 \leq 164 \\
& \quad x_2 \leq 500 \\
& \quad x_1, x_2, x_3 \geq 0
\end{align*}
\]

(b) Identify the decision variables, state the objective function and formulate each constraint.

(c) Interpret the left-hand-side coefficients of each decision variable in part (a) as inputs and outputs of resources per unit activity.

11. Problem 4.23 in Ravindran, p.214


13. Problem 4.15 in Ravindran, p.211

14. Problem 4.16 in Ravindran, p.211

15. Problem 4.17 in Ravindran, p.211

16. For each of the LPs in (a) - (d)

(i) state the corresponding dual problem of the primal as given

(ii) state the two sets of complimentary slackness conditions.

(a)
(b)\[\begin{align*}
\text{max} & \quad 44x_1 - 3x_2 + 15x_3 + 56x_4 \\
\text{s.t.} & \quad x_1 + x_2 + x_3 + x_4 = 20 \\
& \quad x_1 - x_2 \leq 0 \\
& \quad 9x_1 - 3x_2 + x_3 - x_4 \leq 164 \\
& \quad x_1, \ldots, x_4 \geq 0
\end{align*}\]

(c)\[\begin{align*}
\text{max} & \quad 19y_1 + 4y_2 - 8z_2 \\
\text{s.t.} & \quad 11y_1 + y_2 + z_1 = 15 \\
& \quad z_1 + 5z_2 \leq 0 \\
& \quad y_1 - y_2 + z_2 \geq 4 \\
& \quad y_1, y_2 \geq 0
\end{align*}\]

(d)\[\begin{align*}
\text{max} & \quad 10(x_3 + x_4) \\
\text{s.t.} & \quad \sum_{j=1}^{4} x_j = 80 \\
& \quad x_j - 2x_{j+1} \geq 0 \quad j = 1, 2, 3 \\
& \quad x_1, x_2 \geq 0 \quad x_3, x_4 \text{ unspecified}
\end{align*}\]