# Lecture 4 : Circuit Analysis, Resistors - Series/Parallel

**Learning Objectives:**

1. **Combine basic elements to sketch a complete circuit**
2. **Identify series and parallel combination of resistors**
3. **Compute equivalent resistance between two terminals**

## 1. Circuit

An electrical circuit can be seen as an interconnection of electrical elements in a closed path such that current can continuously flow. An example of a circuit is shown below.

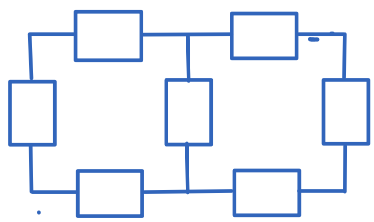


**Fig 4.1 A simple electrical circuit.**

## 2. Node

A point at which two or more elements are connected is called a node. This is illustrated below

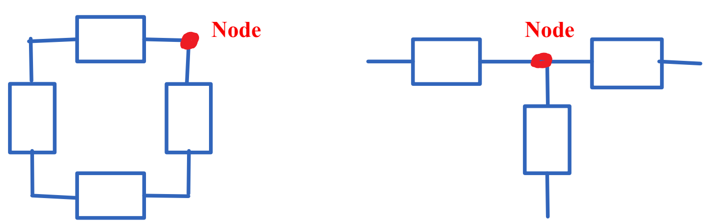



**Fig 4.2 A simple electrical circuit.**

## 3. Loop

Loop is a closed path in a circuit starting at a node, traversing through a series of nodes, and ending at the starting node without passing through the same node twice.

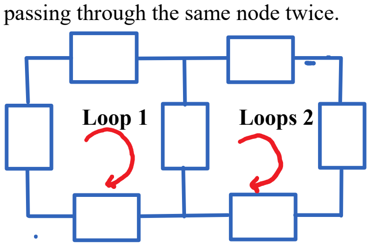



**Fig 4.3 Loops in a circuit**

4. **<u>Series-Connected Elements</u>**

Elements are said to be connected in series if:

1. They share **one** common node.
2. The elements carry the same current.

Figure below illustrates a series connection of elements. One can see that the same current flows in a serial fashion through the elements. It should also be noted that the order of elements does not matter for series connected elements.


**Fig 4.4 Series connected elements. The order of elements can be changed.**

5. **<u>Parallel-connected Elements</u>**

Two elements are connected in parallel if both ends of one element are connected to both ends of another elements by wires. Parallel connected elements have same voltage across them.

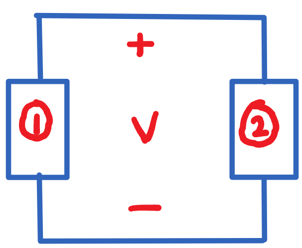



**Fig 4.5 (a)**

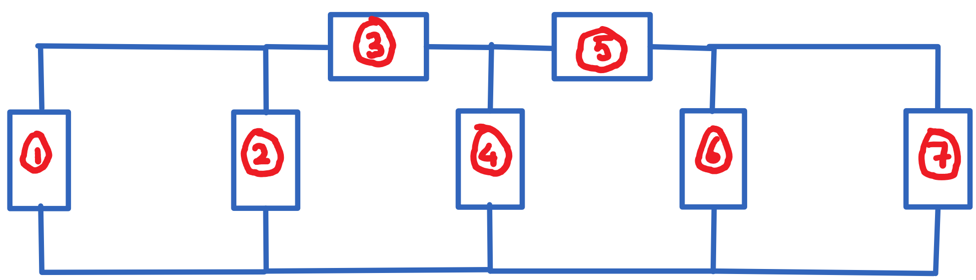



Fig 4.5 (b)

**Fig. 4.5(a) and 4.5 (b) Examples of parallel connected elements.**

**<u>Example:</u>**

Identify parallel connected elements in Fig. 4.5 (a) and Fig. 4.5 (b).

**<u>Solution:</u>**
Fig. 4.5 (a) - Elements 1 and 2.
Fig. 4.5 (b) - Elements 1,2 and Elements 6 and 7.

6. **Series and Parallel connected resistors**

We will now look at series and parallel connection of resistors. Often these resistors can be combined into an equivalent resistance and this can simplify circuit analysis considerably.

## 6.1 Series Connected resistors

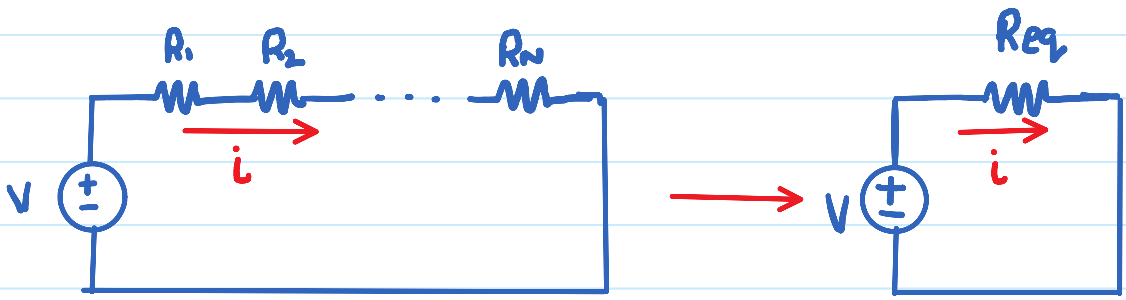



**Fig. 4.6. Series connected resistors**

Fig. 4.6 shows $N$ resistors, $R_1, R_2, \dots, R_N$ connected in series in a circuit. These $N$ resistors can be replaced by an equivalent resistance $R_{eq}$ given by,

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{k=1}^{N} R_k$$

i.e. series connected resistors can be replaced by an equivalent resistance of value equal to sum of the individual resistors. We will prove this in a later lecture when we discuss Kirchhoff's voltage law.

## 6.2 Parallel connected resistors

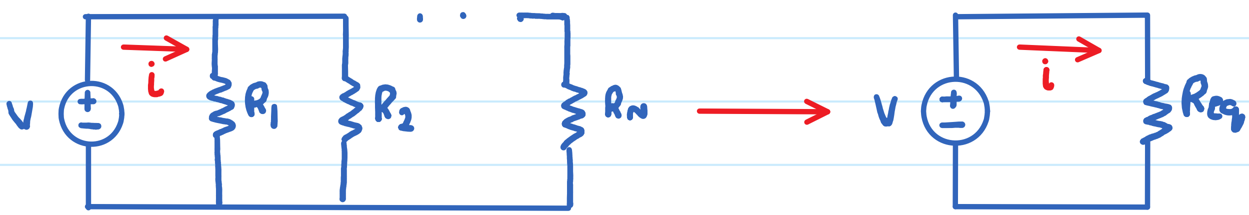



**Fig 4.7. Parallel connected resistors**

Fig. 4.7. shows $N$ resistors, $R_1, R_2, \dots, R_N$, connected in parallel. The parallel resistors can be repaced by an equivalent resistance $R_{eq}$ given by,

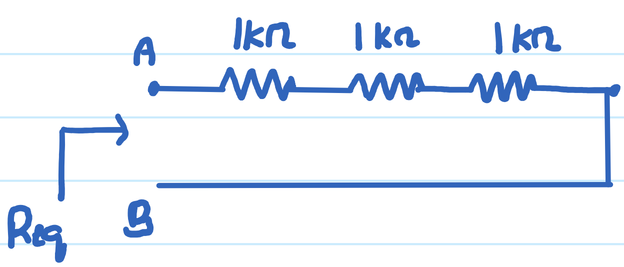
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{k=1}^{N} \frac{1}{R_K}.$$

We will prove this result when we discuss Kirchhoff's current law. For **two resistors**, $R_1$ and $R_2$, in parallel the above expression takes a simpler form.

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}.$$

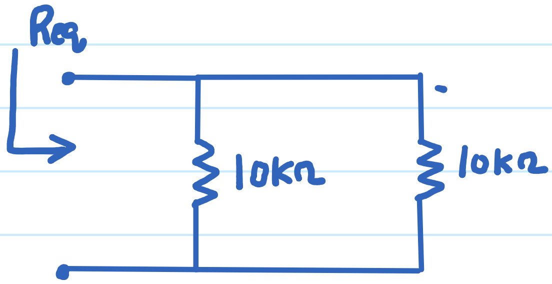
**Examples:**

1. **Compute equivalent resistance between terminals A-B in the circuits shown below**




$R_{eq} = 3\ k\Omega$

**Ex 2.**

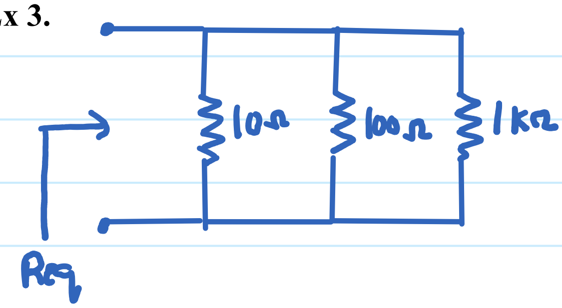



$$R_{eq} = \frac{10 \times 10}{10 + 10} = 5\,k\Omega$$

**Note:** $N$ **identical resistors have a parallel equivalent of**

$$R_{eq} = \frac{R}{N}.$$

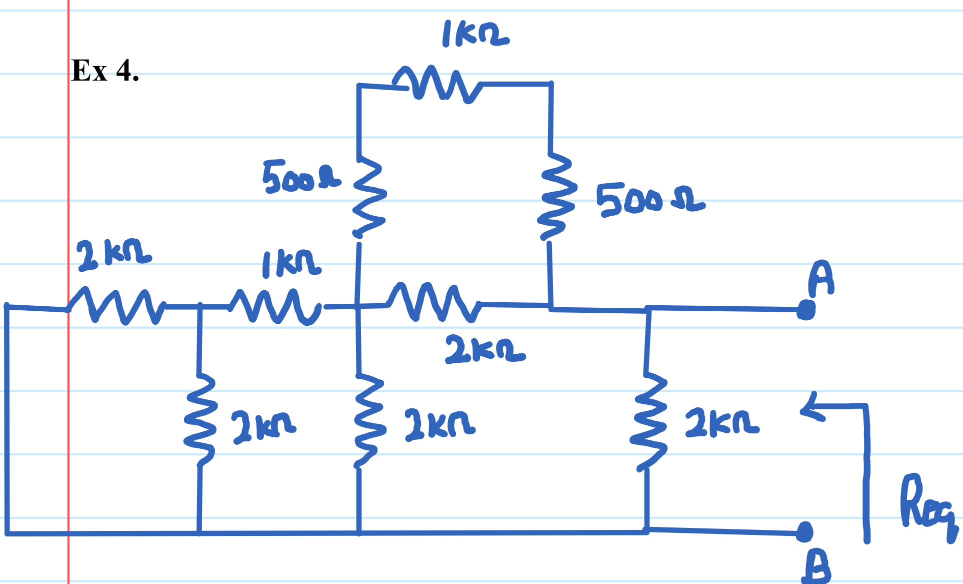
**Ex 3.**




$$R_{eq} = 10 \,//\, 100 \,//\, 1000$$
$$= 9.01\,\Omega$$

**Note: Parallel combination of resistors is dominated by the smallest value.**

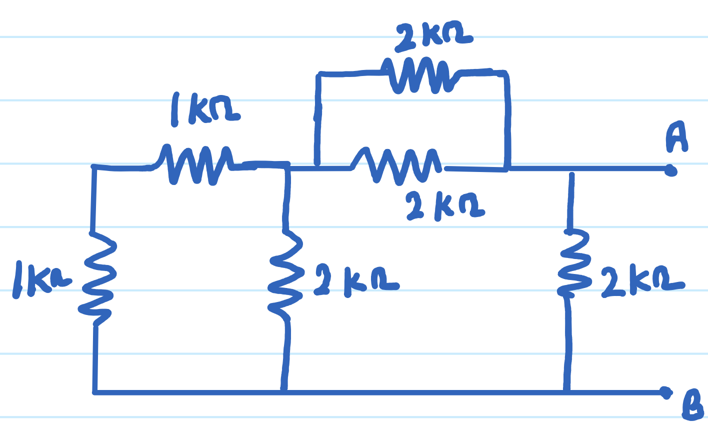
**Ex 4.**



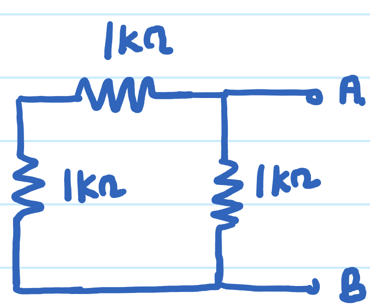

$$R_{eq} = (1+1) \,//\, 2$$

$$R_{eq} = 1\,k\Omega$$

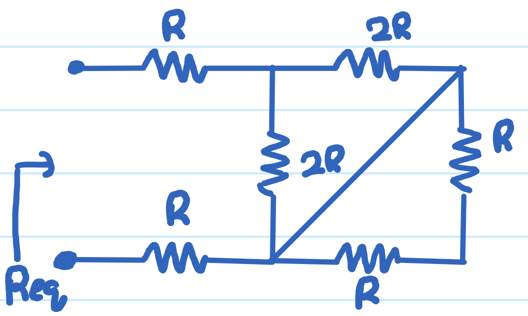
⇒



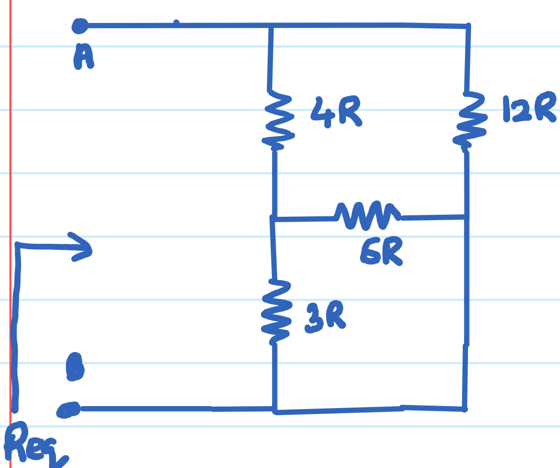

⇒



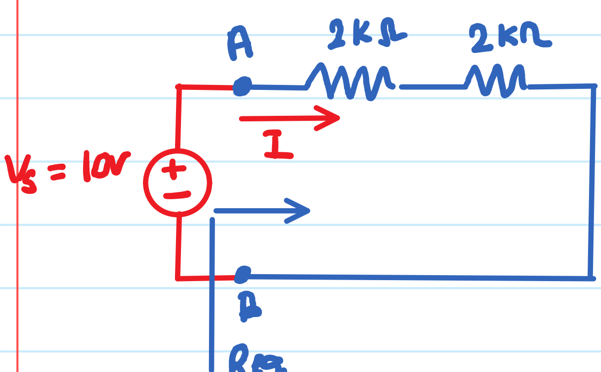
**Ex 5. Solved in class!**

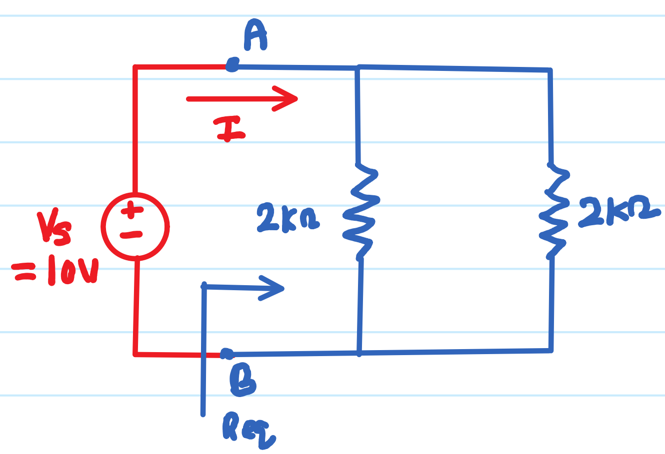



**Ex 6. Solved in class!**

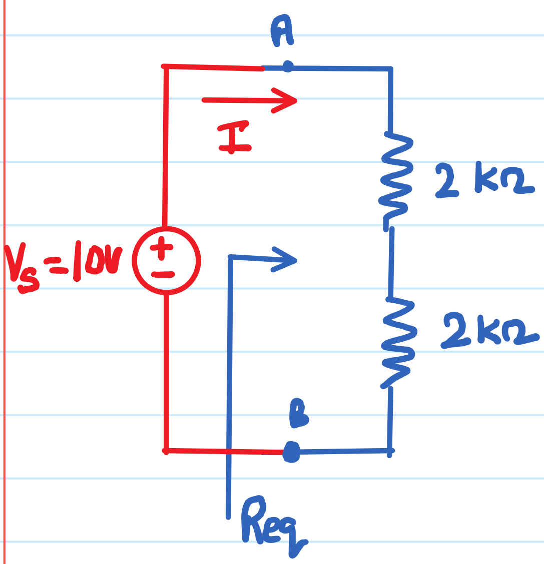


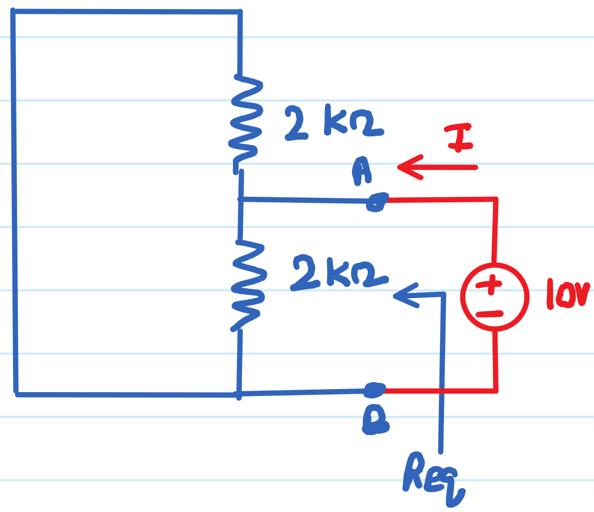

**Ex 7. Find $R_{eq}$ current I in the circuits shown below.**



Ex 8. Find $R_{eq}$ and current I in the circuit shown below.




Ex 9. Find $R_{eq}$ and current I in the circuit shown below.